THE 2016 GAINESVILLE NUMBER THEORY CONFERENCE

ALLADI 60

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ABSTRACTS

S. D. Adhikari, Harish-Chandra Institute

Some weighted zero-sum results

We shall have a glimpse of some generalizations of a classical zero-sum result and shall describe some recent progress related to one of the weighted generalizations.

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Scott Ahlgren, University of Illinois

Kloosterman sums and Maass cusp forms of half integral weight for the modular group

Kloosterman sums appear in many areas of number theory. We estimate sums of Kloosterman sums of half-integral weight on the modular group. Our estimates are uniform in all parameters in analogy with Sarnak and Tsimerman's improvement of Kuznetsov's bound for the ordinary Kloosterman sums. As an application, we obtain an improved estimate for the classical problem of estimating the size of the error term in Rademacher's famous formula for the partition function. This is joint work with Nick Andersen.

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Nickolas Andersen, University of Illinois

Algebraic and transcendental formulas for the smallest parts function

We study the smallest parts function $\mathrm{spt}(n)$ introduced by Andrews. The generating function for $\mathrm{spt}(n)$ forms a component of a natural mock modular form of weight $3/2$ whose shadow is the Dedekind eta function. We obtain two formulas for $\mathrm{spt}(n)$ which are analogues of the formulas of Rademacher and Bruinier-Ono for the ordinary partition function. The convergence of our expression is non-trivial; the proof relies on a power savings estimate for weighted sums of Kloosterman sums of half-integral weight which follows from spectral methods. This is joint work with Scott Ahlgren.

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George Andrews, Pennsylvania State University

On the Alladi-Schur Theorem

In 1926, I. Schur proved that if $A(n)$ equals the number of partitions of n into parts congruent to 1 or 5 modulo 6, and $B(n)$ equals the number of partitions of n in which any two parts differ by at least 3 and multiples of 3 differ by more than 3, then $A(n) = B(n)$. In the 1990's, K. Alladi noted that if $C(n)$ equals the number of partitions of n into odd parts none repeated more than twice, then also $C(n) = B(n)$. The talk begins with a speculative history on why Schur found $A(n) = B(n)$ but not $B(n) = C(n)$. We then consider the following refinement of the Alladi-Schur theorem:
THEOREM. Let $C(m, n)$ denote the number of partitions among those enumerated by $C(n)$ that have exactly m parts. Let $B(m, n)$ denote the number of partitions among those enumerated by $B(n)$ where the number of odd parts plus twice the number of even parts equals m. The $B(m, n) = C(m, n)$.
We conclude with reasons for believing $B(n) = C(n)$ to be the most natural formulation of Schur's theorem and how this holds implications for future research.

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Richard Askey, University of Wisconsin

Some inequalities for symmetric polynomials

There will be two topics in this talk. The first will deal with a problem when teaching calculus. After introducing Rolle's theorem and giving some reasons why it is true, it is used to prove the mean value theorem. Then Rolle's theorem is basically ignored since the mean value theorem is used instead. When I taught calculus this is what I did, since the only other direct uses of Rolle's theorem I knew involved problems which would take too long to motivate for students just starting calculus. I now know a problem which can be solved with Rolle's theorem and little else which is easy to motivate. Part of the surprise comes from when the inequalities which arise in this problem were first proven.
The second type of inequality which will be discussed seems to have first arisen in 1820 by Lehmus, but most of the references to this inequality are to an inequality found by Schur which Hardy, Littlewood and Polya included as a problem in their book "Inequalities". Some related results will be given.

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Roger Baker, Brigham Young University

Limit points and long gaps between primes

Let d_n denote the nth prime gap. Recent progress in the study of normalized prime gaps has led to the conclusion that the sequence $d_n/f(n)$ has limit points occupying at least 25% of the positive real line, if $f(n)$ is a well-behaved function that grows to infinity no faster than $R(n) = \log n \log_2(n) \log_4(n)/(\log_3(n))^2$. This is due to Pintz, building on work of Banks,Freiberg and Maynard. The speaker and Tristan Freiberg have obtained the corresponding result with $R(n)$ replaced by any well-behaved function growing slower than $R(n) \log_3(n)$, the function occurring in the recent gap result of Ford, Green, Konyagin, Maynard and Tao. Our work draws on the techniques in the three works cited above, especially the last one. The talk covers some indications of these techniques .

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Alexander Berkovich, University of Florida

Taking a trip down memory lane

In my talk I review my joint work with Professor Alladi. In particular, I will discuss our joint work on the Schur and Göllnitz partition theorems. I will also unveil some new exciting results inspired by Alladi's research on weighted partitions.

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Bruce Berndt, University of Illinois

Integrals Associated with Ramanujan and Elliptic Functions

We evaluate in closed form certain classes of infinite integrals containing trigonometric and hyperbolic trigonometric functions in their integrands. Although not apparent, the integrals are related to Jacobian elliptic functions, in particular, to theorems found in Ramanujan's notebooks. One of our evaluations answers a question first posed by M. E. H. Ismail.

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Edward Bertram, University of Hawaii at Manoa

Recent progress on an old unsolved problem in finite group theory

In 1903 E.Landau showed why, given a positive integer k, only a finite number of finite groups have exactly k conjugacy classes. In the 1960's several authors (including P. Erdős and P.Turan) proved, using only the class-equation, that always $k(G) > \log_2 \log_2(|G|)$. But group-theorists have also proved that for infinite collections of groups, including all supersolvable groups, that $k(G) > \log_3 |G|$, and from the contributions of many authors toward classifying finite groups by their number k of classes (now completed for $k < 15$), we know that whenever $|G|$ is no more than 3^{15}, $k(G) > \log_3(|G|)$. When G is a solvable group, we discuss number-theoretic evidence related to the prime power factorization of $|G|$, which along with recent developments would yield that $k(G) > \log_3(|G|)$ when G is solvable and $|G|$ is sufficiently large.

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Manjul Bhargava, Princeton University

Squarefree values of polynomial discriminants

The question as to whether a positive proportion of monic integer polynomials of degree n have squarefree discriminant is an old one; an exact formula for the density was conjectured by Lenstra. (The interest in polynomials f with squarefree discriminant comes from the fact that in such cases it is immediate to construct the ring of integers in the $\mathbb{Q}$-algebra $\mathbb{Q}[x]/f(x)$.)
In this talk, we will describe recent work with Arul Shankar and Xiaoheng Wang that allows us to determine the probability that a random monic integer polynomial has squarefree discriminant - thus proving the conjecture of Lenstra.

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Gaurav Bhatnagar, University of Vienna

Heine's method and A_n generalisations of Ramanujan's transformation formulas

We apply an idea originally due to Heine, and explained by Andrews and Berndt, to study transformation formulas for A_n basic hypergeometric series. As special cases we find some multiple series generalisations of transformation formulas due to Ramanujan. In many cases, our formulas are very close to Ramanujan's identities. Our work uses multivariable identities due to Gustafson and Krattenthaler (1997), Kajihara (2004), Milne (1997) and Milne and Lilly (1995).

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Kathrin Bringman, University of Cologne

Ramanujan and meromorphic modular forms

Much is known about Fourier coefficients of weakly holomorphic modular forms, due to the famous formulas of Hardy, Ramujan and others. In this talk, I will report on coeffiicents of meromorphic modular forms, allowing poles in the upper half-plane. This is all joint work with Ben Kane.

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Dale Brownawell, Pennsylvania State University

Divided Derivatives of the Carlitz Period

Without telling anyone what he was doing, Carlitz defined an analogue of the exponential function and thus of π in the function field setting over a finite field. L. Denis showed the algebraic independence of this π and its first $p-1$ derivatives. In work begun already with A.J. van der Poorten, we show the algebraic independence of all divided derivatives of this π. Very recently announced work of M. Papanikolas expresses the fundamental periods of the Anderson-Thakur tensor powers of the Carlitz module in terms of these divided derivatives.

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Neil Calkin, Clemson University

What Moser Could Have Asked: Hamilton Cycles in Tournaments

Moser asked for a construction of explicit tournaments on n vertices having at least $(\frac{n}{3e})^n$ Hamilton cycles. We show that he could have asked for rather more.

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Song Heng Chan, Nanyang Technological University, Singapore

Truncated series from the Quintuple Product Identity

Recently G.E. Andrews and M. Merca made a conjecture on truncated series related to the Jacobi Triple Product Identity. V.J.W. Guo and J. Zeng also made a similar conjecture. This conjecture was later proved independently by A. J. Yee and R. Mao. In this talk, we examine the next set of truncated series, i.e., truncated series derived from the quintuple product identity. We prove that one has nonnegative coefficients and the other has nonpositive coefficients. In addition, we show that truncated series arising from two well-known consequences of the quintuple product identity also have such nonnegativity. This is a joint work with Renrong Mao and Ho Thi Phuong Nhi.

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Shaun Cooper, Massey University, New Zealand

Holonomic Alchemy and Series for $1/\pi$

The theory of Ramanujan's series for $1/\pi$ received a boost with the announcement on the `arXiv.org` of a large number of conjectures by Z.-W. Sun during 2011–2014. This talk will discuss several of the conjectures and their proofs. Several issues that remain unresolved will be outlined. This is joint work with Wadim Zudilin.

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Hédi Daboussi, Université de Picardie, Amiens,France

On arithmetic functions

We survey some results obtained by dealing with smooth (or friable) numbers and rough numbers.

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Jean-Marc Deshouillers, Bordeaux INP

Sarnak's conjecture and automatic sequences

We present a joint work with Michael Drmota and Clemens Muellner (Vienna). Sarnak's conjecture (in short, concerning some indepence between the Moebius function and any other kind of naturally defined sequence) should hold for deterministic sequences, among other the so-called "automatic sequences". We indeed show that this is the case for almost all the automatic sequences, namely those which are produced by a "synchronizing" automaton. We'll give also some results concerning some other cases.

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Harold Diamond, University of Illinois

PNT Equivalences and Nonequivalences for Beurling primes

In classical prime number theory there are several asymptotic formulas that are called "equivalent" to the PNT. We show conditions under which analogues of these relations do or do not hold for Beurling generalized numbers.

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Atul Dixit, Indian Institute of Technology, Gandhinagar

Overpartition analogues of partitions associated with the Ramanujan/Watson mock theta function $\omega(q)$

Let $\omega(q)$ denote a third order mock theta function of Ramanujan and Watson. Recently George E. Andrews, Ae Ja Yee and I showed that $q\omega(q)$ is the generating function of $p_\omega(n)$, the number of partitions of a positive integer n such that all odd parts are less than twice the smallest part. We also studied the associated smallest parts partition function $\mathrm{spt}_\omega(n)$ and proved some congruences for the same. In this talk, we will discuss the overpartition analogue of $p_\omega(n)$, namely, $\overline{p}_\omega(n)$. Finding an alternate representation for the generating function of $\overline{p}_\omega(n)$ turns out to be difficult in this case. We devise a new seven parameter q-series identity which generalizes a deep identity of Andrews (as well as its generalization by R. P. Agarwal), and then specialize it, along with the use of some identities in basic hypergeometric series, to arrive at an alternate representation in terms of a ${}_3\phi_2$ and an infinite series involving the little q-Jacobi polynomials. We also prove some congruences for $\overline{p}_\omega(n)$ and for the overpartition analogue of $\mathrm{spt}_\omega(n)$. This is joint work with George E. Andrews, Daniel P. Schultz and Ae Ja Yee.

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Jehanne Dousse, Universität Zürich

The method of weighted words and a refinement of Siladić's theorem

Alladi and Gordon introduced the method of weighted words in 1993 to find refinements of Schur's theorem. It was later used to find refinements of Göllnitz' theorem and Capparelli's theorem, a partition identity which arose in the study of Lie algebras. In this talk, we will show how the method of weighted words combined with q-difference equations and recurrences allows to find a non-dilated version and refinement of Siladić's identity, which also comes from Lie algebras theory.

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Dennis Eichhorn, University of California, Irvine

A New Decomposition of Partitions and its Arithmetic Implications

In 1990, Garvan, Kim, and Stanton used the decomposition of a partition into its ℓ-core and ℓ-quotient to give explicit cycles of partitions which give direct concrete combinatorial proofs of the first few Ramanujan congruences for $p(n)$. Recently, Breuer, Kronholm, and the speaker discovered a new decomposition of a partition into its "ℓ-box remainder" and "ℓ-box quotient" which allows for the construction of new cycles which provide direct concrete combinatorial proofs of both congruences and the periodicity of $p(n, d)$, the number of partitions of n into parts of size at most d, modulo M. In this talk, we discuss these decompositions, the cycles of partitions they generate, and the arithmetic implications.

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Ali-Bulent Ekin, Ankara University

Some relations among the components of the partition generating function

Partition Generating function is

$$\sum_{n=0}^{\infty} p(n)q^n = \prod_{n=1}^{\infty}(1-q^n)^{-1},$$

where $p(n)$ denotes the number of partitions of nonnegative integer n. Components are defined

$$\mathbf{P}^{(r)} := q^r \sum_{n=0}^{\infty} p(mn+r)q^{mn},$$

where $m \geq 2$ is positive integer and $r = 0, 1, 2, \ldots, m-1$. Kolberg gave interesting relations among $\mathbf{P}^{(r)}$ when $m = 2, 3, 5$ and 7. Following Kolberg here we study the components when $m = 11$ and 13.

Atkin and Swinnerton-Dyer's method is based on expressing a power series as a polynomial whose coefficients are also power series. We follow the same way to get the components of the generating function of partitions in the cases mod 11 and 13. But, these are to be simplified to get a simple form and the congruence properties of the components. We explain how a certain subgroup of $\mathrm{SL}_2(\mathbb{Z})$ acts on the components.

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Peter Elliott, University of Colorado, Boulder

From Ramanujan to the theory of groups, a particular history of multiplicative functions

Beginning with the abstract theory of multiplicative functions and their connection to two apparently unconnected disciplines catalysed by Ramanujan... .

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Hershel Farkas, The Hebrew University of Jerusalem

The Schottky Relation in arbitrary genus

We shall indicate how to write down a version of the Schottky relations in arbitrary genus which agree with the Poincare relations near the diagonal and therefore are functionally independent there.

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Amanda Folsom, Amherst College

Zeros of modular forms of half integral weight

We study canonical bases for spaces of weakly holomorphic modular forms of level 4 and weights in $\mathbb{Z} + \frac{1}{2}$, and show that almost all modular forms in these bases have the property that many of their zeros in a fundamental domain for $\Gamma_0(4)$ lie on a lower boundary arc. We also give applications to the mock modular generating function for Hurwitz class numbers, and to the simultaneous non-vanishing of associated cusp form coefficients. This is joint work with Paul Jenkins (Brigham Young University).

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Kevin Ford, University of Illinois

Large gaps between primes and related problems

We describe recent improvements to lower bounds on the largest gap between consecutive primes less than x. This is joint work with Ben Green, Sergei Konyagin, James Maynard and Terence Tao. We also describe work on the problem of finding long strings of composite numbers containing an integers of a prescribed type, such as a perfect k-th power (joint with Sergei Konyagin and Roger Heath-Brown).

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Dorian Goldfeld, Columbia University

On an additive prime counting function of Alladi and Erdős

Let $n = \prod_{i=1}^{r} p_i^{a_i}$ be the unique prime decomposition of a positive integer n. In 1977, Alladi and Erdős introduced the additive function

$$A(n) := \sum_{i=1}^{r} a_i p_i.$$

Among several other things they proved that $A(n)$ is uniformly distributed modulo 2. In this talk we will show that $A(n)$ is uniformly distributed module q for any integer $q \geq 2$.

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Dan Goldston, San Jose State University

Sums and Differences of Pairs of Primes

This talk will examine some problems concerned with sums and differences of pairs of primes. One specific problem we will discuss concerning sums of two primes is the average number of Goldbach representations for all the integers up to N. This is joint work with Yang Liyang. Much work has been done on problems concerning the difference between consecutive primes, but it is also interesting to consider the differences between pairs of primes which may or may not be consecutive. One problem here is to look at the primes up to N and try to determine the most frequent difference that occurs. This problem can be solved assuming a sufficiently strong form of the Hardy-Littlewood prime pair conjecture. One can obtain good numerical evidence for the answer and also prove interesting unconditionally results for this problem. This is joint work with S. Funkhouser, D. Sengupta, and J. Sengupta.

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Ron Graham, University of California San Diego

Monochromatic solutions to linear equations

In this talk I will describe a number of problems involving the existence and enumeration of solutions to a variety of (systems of) linear equations. Most of the questions generated are still unresolved.

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Michael Griffin, Princeton University

p-adic harmonic Maass forms

Harmonic Maass forms posses many intricate p-adic properties. Guerzhoy, Kent, and Ono, and others have extensively studied the p-adic properties of the q-series of related mock modular forms. Special values of Harmonic Maass forms also possess similarly interesting p-adic properties.

In his doctoral thesis, Candelori investigated a theory of integer weight *p-harmonic Maass forms* arising from the de Rham cohomology for p-adic modular forms. We consider a similar theory, although our approach and definitions differ somewhat from Candelori's. We construct p-adic analogues of classical harmonic Maass forms of weight 0 and $1/2$ with square free level by means of the Hecke algebra. As in the classical case these forms are connected to positive weight modular forms by certain differential operators. Moreover, the coefficients of the half integral weight forms can be given as modular traces of weight 0 forms over Heegner divisors. The complex harmonic Maass forms and their corresponding p-adic analogues may also be collected into an adelic theory.

As an application, we consider elliptic curves $E/\mathbb{Q}$ with square free conductor. Building on work of Bruinier and Ono, we construct a function H'_E, whose vanishing at Heegner points determines the vanishing of central L derivatives of quadratic twists of E.

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Pavel Guerzhoy, University of Hawaii

Half-integral weigh p-adic coupling

For a weakly holomorphic Hecke eigenform of even integral weight, one applies Atkin's U-operator repeatedly. The p-adic limit exists as a q-series and can be identified with a certain holomorphic Hecke eigenform. We present a parallel result which requires a very different proof for a half-integral weight eigenform. This is a joint project with K. Bringmann and B. Kane.

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Mike Hirschhorn, University of New South Wales, Australia

Ramanujan's tau function

Ramanujan's tau function is defined by

$$\sum_{n\geq 1}\tau(n)q^n = q\prod_{n\geq 1}(1-q^n)^{24}.$$

The tau function has many fascinating properties. One of these is that for prime p,

$$\tau(pn) = \tau(p)\tau(n) - p^{11}\tau\left(\frac{n}{p}\right), \tag{1}$$

where it is understood that $\tau\left(\frac{n}{p}\right) = 0$ if $p \nmid n$.

I have recently managed to give proofs of (1) for $p = 2,\ 3,\ 5$ and 7 which require nothing more than Jacobi's triple product identity,

$$\prod_{n\geq 1}(1+a^{-1}q^{2n-1})(1+aq^{2n-1})(1-q^{2n}) = \sum_{-\infty}^{\infty} a^n q^{n^2}.$$

I will present one or more of these proofs.

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Tim Huber, University of Texas - Rio Grande Valley

Higher level Ramanujan-Sato series for $1/\pi$

A systematic construction for Ramanujan-Sato expansions from McKay-Thompson series is given. Expansions for each divisor of the order of the Monster are derived, and a uniform interpretation is given for series parameters as generators of invariant function fields for subgroups of $\Gamma_0(n)$. Relations between the generators extend reciprocal identities satisfied by eta quotients and the continued fractions of Rogers-Ramanujan and Göllnitz-Gordon. Complete lists of rational and quadratic series are derived from singular values of the parameters. Heuristics will be given to minimize the order of recurrences defining the series expansions. This is joint work with Daniel Schultz and Dongxi Ye.

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Mourad Ismail, University of Central Florida

Identities related to the Rogers-Ramanujan identities

We introduce a generalization of the Ramanujan function and give an extension of the m-version of the Rogers-Ramanujan identities. In addition we give several Rogers-Ramanujan type identities.

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Aleksandar Ivic, Serbian Academy of Sciences

Hardy's function Z(t) - results and problems

Hardy's function

$$Z(t) := \zeta(1/2+it)\chi^{-1/2}(1/2+it), \zeta(s) = \chi(s)\zeta(1-s)$$

plays an important role in the detection of zeros of the Riemann zeta-function on the critical line $\Re s = 1/2$. The talk will present some recent results and problems on $Z(t)$. These primarily involve moments of $Z(t)$ and the distribution of its values.

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Marie Jameson, University of Tennessee

On p-adic modular forms and the Bloch-Okounkov theorem

Bloch-Okounkov studied certain functions on partitions f called shifted symmetric polynomials. They showed that certain q-series arising from these functions (the so-called *q-brackets* $\langle f\rangle_q$) are quasimodular forms. We revisit a family of such functions, denoted Q_k, and study the p-adic properties of their q-brackets. To do this, we define regularized versions $Q_k^{(p)}$ for primes p. We also use Jacobi forms to show that the $\langle Q_k^{(p)}\rangle_q$ are quasimodular and find explicit expressions for them in terms of the $\langle Q_k\rangle_q$.

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Min-Joo Jang, University of Cologne

On spt-crank type functions

In a recent paper, Andrews, Dixit, and Yee introduce a new spt-type function $\mathrm{spt}_\omega(n)$, which closely related with Ramanujan's third order mock theta function $\omega(q)$. Garvan and Jennings-Shaffer introduce a crank function which explain congruences for $\mathrm{spt}_\omega(n)$. In this talk, we study asymptotic behavior of this crank function and confirm a positivity conjecture of the crank asymptotically. We also study a sign pattern of the crank and congruences for $\mathrm{spt}_\omega(n)$. This is joint work with Byungchan Kim.

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Chris Jennings-Shaffer, Oregon State University

Partition Rank Functions Transforming Like Their Partition Functions

Recently Garvan revisited the transformation properties due to Bringmann and Ono for the rank of partitions as a mock modular form. The associated harmonic Maass form was found to transform like the partition function on a certain subgroup of the modular group. We show that this also occurs with the Dyson rank of overpartitions and the M2-rank of partitions without repeated odd parts transforming like the generating functions for overpartitions and partitions without repeated odd parts. We briefly discuss the utility of this fact in calculating identities involving partition ranks.

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Abhas Kuma Jha, NISER, Bhubaneswar, India

Rankin-Cohen brackets on Siegel modular forms and special values of certain Dirichlet series

Given a fixed Siegel cusp form of genus two, we consider a family of linear maps between the spaces of Siegel cusp forms of genus two by using the Rankin-Cohen brackets and then we compute the adjoint maps of these linear maps with respect to the Petersson scalar product. The Fourier coefficients of the Siegel cusp forms of genus two constructed using this method involve special values of certain Dirichlet series of Rankin type associated to Siegel cusp forms. This is a joint work with B. Sahu.

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Ben Kane, University of Hong Kong

Inner products of meromorphic modular forms

In this talk, we discuss the computation of inner products of arbitrary meromorphic modular forms against a special fixed meromorphic modular form constructed via integral binary quadratic forms of a fixed (negative) discriminant. In particular, the inner product between two of these special "quadratic form" meromorphic forms is related to higher Green's functions evaluated at CM-points. This is joint work with Kathrin Bringmann and Anna von Pippich.

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K. Kannan, SASTRA University

Development of Gravitational Search Algorithm for solving a class of Ternary Diophantine Equations

The paper introduces Gravitational Search Algorithm (GSA) to solve some special forms of Ternary Diophantine equation, for which there exists no general method of finding solutions. This algorithm is found upon introducing randomization concept along with the two of the four primary parameters 'velocity' and 'gravity' in physics. The performance of this algorithm has been evaluated on a set of random values. Computational result shows that the gravitational search algorithm - based heuristic is capable of producing high quality solutions, can offer many solutions of such equations. This work is joint with S. Raja Balachandar, R. Srikanth, S. Balachandran and S. G. Venkatesh.
Keywords: Gravitational Search Algorithm, Ternary Diophantine Equation, Randomization, primary parameters.

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Kevin Keating, University of Florida

Extensions of local fields and elementary symmetric polynomials

Let K be a local field with valuation v_K. Let K^{sep} be a separable closure of K, and let L/K be a finite totally ramified subextension of K^{sep}/K of degree n. Let $\sigma_1, \ldots, \sigma_n$ denote the K-embeddings of L into K^{sep}. For $1 \leq i \leq n$ let $s_i(X_1, \ldots, X_n)$ denote the ith elementary symmetric polynomial in n variables, and for $\alpha \in L$ set $S_i(\alpha) = s_i(\sigma_1(\alpha), \ldots, \sigma_n(\alpha))$. In this talk we consider the problem of finding a lower bound for $v_K(S_i(\alpha))$ in terms of $v_L(\alpha)$. The solution seems to depend on the indices of inseparability of L/K.

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Byungchan Kim, Seoul National University of Science and Technology

Partial theta functions and partial indefinite theta series

K. Alladi has studied extensively a partial theta function identity in the lost notebook. After a brief survey on how my work is related with Alladi's, I am going to give an application of partial theta function identities to unimodal ranks. I am also going to introduce partial indefinite theta series identities based on a recent joint work with J. Lovejoy.

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Susie Kimport, Stanford University

The asymptotics of partial theta functions

Partial theta functions are sums whose terms resemble those of modular theta functions, save that the sums are taken over an incomplete lattice. In one of his notebooks, Ramanujan wrote down an asymptotic expansion for one particular partial theta functions as $q \to 1$. In 2011, Berndt and Kim generalized this type of asymptotic expansions to a related family, still for $q \to 1$. In this talk, we will extend the asymptotic results of Berndt and Kim to the case of $q \to e^{2\pi ih/k}$, any root of unity. In addition, we will present new asymptotic expansions of another family of partial theta functions.

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Sun Kim, University of Illinois

Bressoud's Conjecture

By employing Andrews' generalization of Watson's q-analogue of Whipple's theorem, D. M. Bressoud obtained an analytic identity, which specializes to most of the well known theorems on partitions with part congruence conditions and difference conditions. This led him to define two partition functions A and B depending on multiple parameters as combinatorial counterparts of his identity. Bressoud then proved that $A = B$ for $\lambda = 0, 1$, and $\lambda = k = r = 2$, and conjectured that $A = B$ holds true for any $k \geq r \geq \lambda \geq 2$. In this talk, we discuss Bressoud's conjecture for even λ.

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Christian Krattenthaler, University of Vienna

Discrete analogues of Macdonald-Mehta integrals

I shall consider certain discretisations of the Macdonald-Mehta integrals. There are ten families of such discretisations which can be evaluated in closed form. I shall sketch the ideas which go into the proofs of these identities, which come from identities for classical group characters, combinatorics of non-intersecting lattice paths, and a transformation formula for elliptic hypergeometric series, respectively. This is joint work with Richard Brent and Ole Warnaar.

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Brandt Kronholm, University Of Texas Rio Grande Valley

Quasipolynomials, Polyhedral Geometry, Divisibility Patterns and Combinatorial Witnesses for Partitions

In this presentation, we discuss the quasipolynomial for the restricted partition function $p(n, m)$ which enumerates the number of partitions of n into exactly m parts. We show that it treats both divisibility properties of these restricted partition numbers but also encodes a combinatorial witness to these divisibilities.

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Kagan Kursungoz, Sabanci University, Istanbul

An Open Question of Corteel, Lovejoy and Mallet

Finding overpartition analogues of partition identities has partly been focus of recent research. Rogers-Ramanujan generalizations, not surprisingly, occupy central stage. In the concluding remarks in their paper "An extension to overpartitions of the Rogers-Ramanujan identities for even moduli" (JNT 128, 2008 pp 1602-1621), Corteel et.al. gave a general series for further investigation. We settle the question, unfortunately, in the negative. We will place their series in a family of series, and show that one cannot deduce further overpartition identities than already proven. This is joint work with Shashank Kanade (Univ of Alberta) and Matthew Russell (Rutgers Univ).

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Robert Lemke Oliver, Stanford University

Consecutive primes modulo q

The primes are well-known to be equidistributed among the admissible residue classes modulo any integer $q \geq 3$. Here, we consider the distribution of consecutive primes modulo q, and we find considerable deviation from the natural prediction. We offer a conjectural explanation for this deviation, and we provide a conditional proof of this conjecture. This is joint work with Kannan Soundararajan.

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Jim Lepowsky, Rutgers University

"Motivated proofs" of generalized Rogers-Ramanujan identities as stimuli for new ideas in vertex operator algebra theory

Some years ago, G. Andrews and R. Baxter gave what they termed a "motivated proof" of the Rogers-Ramanujan identities. This proof, indeed "motivated," on the one hand was in fact a variant of certain proofs of Rogers and Ramanujan, and on the other hand seemed to me to suggest new hidden structure in the ongoing vertex-operator-algebraic approaches to the Rogers-Ramanujan identities and generalizations. I'll survey a number of recent developments on "motivated proofs" of families of partition identities of Rogers-Ramanujan type, including work of B. Coulson, S. Kanade, R. McRae, F. Qi, M. Russell, S. Sadowski, A. Sills, C. Takita, M. Zhu and myself. Such proofs are expected to play an important role in the long-term interaction between the representation theory of vertex operator algebras and the theory of partitions. Recently, a new "conceptual" approach to such "motivated proofs" is being developed by my student Bud Coulson. I'll sketch these ideas.

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Winnie Li, Pennsylvania State University

Combinatorial zeta and L-functions

Zeta functions are counting functions in nature. For instance the Dedekind zeta function for a number field counts integral ideals, the zeta function for a variety defined over a finite field counts solutions in finite extensions of the base field, while the Selberg zeta function counts closed geodesics in a compact Riemann surface. In this talk we shall discuss the analogue of zeta and L-functions in combinatorial setting, that is, attached to graphs and higher dimensional simplicial complexes. Similarities and dissimilarities between number theoretic and combinatorial zeta functions will also be compared.

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Zhi Guo Liu, East China Normal University, Shanghai

A system of q-partial differential equations and q-polynomials

We introduce a system of q-partial differential equations and prove that if an analytic function in several variables satisfies these q-partial differential equations, then, it can be expanded in terms of the product of some homogeneous polynomials. A few applications of this expansion theorem to q-series are discussed.

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Steffen Loebrich, University of Cologne

Poincaré Series and Traces of Singular Moduli

We write Fourier coefficients of certain weight 2 Poincaré series recently investigated by Bringmann and Kane in terms of Faber polynomials of the j-function. By summing over CM-points of binary quadratic forms of negative discriminant, we can interpret traces of singular moduli in the context of almost holomorphic modular forms of weight 2. This new setting allows us to find a different approach to explicit series expressions for traces of singular moduli.

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Lisa Lorentzen, Norwegian University of Science and Technology

Convergence of random continued fractions

It is amazing to see how willingly a continued fraction converges. It has therefore been a dream of mine to prove that "almost all continued fractions converge" in some sense. Now, a continued fraction can be regarded as a sequence of Möbius transformations, and thus, as a sequence of non-singular (2×2) matrices. So if we define a *random continued fraction* $K(a_n/b_n)$ as a stochastic variable where the elements (a_n, b_n) are picked independently from a given distribution μ, we can adapt the theory of random products of matrices to prove convergence of $K(a_n/b_n)$ with probability 1. Of course, there are conditions on the measure, but it turns out that a finite expectation of the expression $\log(|a_1| + (1 + |b_1|^2)/|a_1|)$ plus some extra very mild technical conditions suffices. I have talked on this topic on different occasions. The new thing in this talk is that we are interested in the speed of convergence that can be obtained with probability 1.

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Karl Mahlburg, Louisiana State University

Alladi, Schur's second partition theorem, and universal mock theta functions

I will discuss old and new results related to Alladi's work on Schur's second partition theorem, in which he notably developed the method of Weighted Words in order to prove analytic and combinatorial partition identities. Among the recent developments is a striking connection between Schur's theorem, Alladi's q-continued fraction, and Hickerson's universal mock theta function.

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Helmut Maier, Universität Ulm

Recent Results on Gaps between Primes (joint with Michael Th. Rassias)

The authors combine a recent improvement by K.Ford,B.J.Green, S.Konyagin, J.Maynard and T.Tao of results of Erdős and Rankin on large gaps between consecutive primes with a result of K.Ford, D.R.Heath-Brown and S.Konyagin. They establish the existence of infinitely many large gaps between primes containing k-th powers of primes.

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Bibekananda Maji, Harish-Chandra Research Institute, Allahabad

Modular type relations associated to the Rankin-Selberg L-function

Hafner and Stopple proved a conjecture of Zagier relating to the asymptotic behaviour of the inverse Mellin transform of the symmetric square L-function associated to Ramanujan tau function. In this paper we prove similar result for any cusp form over the full modular group.

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Amita Malik, University of Illinois

Partitions into kth powers of terms in an arithmetic progression

G. H. Hardy and S. Ramanujan established an asymptotic formula for the number of unrestricted partitions of a positive integer, and claimed a similar asymptotic formula for the number of partitions into perfect kth powers, which was later proved by E. M. Wright. Recently, R. C. Vaughan gave a simpler asymptotic formula in the case $k = 2$. We study partitions into parts from a specific set $A_k(a_0, b_0) := \left\{m^k : m \in \mathbb{N}, m \equiv a_0 \pmod{b_0}\right\}$, for fixed positive integers k, a_0, and b_0, and obtain an asymptotic formula for the number of such partitions. This is a generalization of the earlier mentioned results by Wright and Vaughan. We also discuss the parity of the number of these partitions. This is joint work with Bruce Berndt and Alexandru Zaharescu.

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James Maynard, University of Oxford

The distribution of prime numbers

Talk 1 (Ramanujan Colloquium): Linear equations in primes
Many of the most famous and most important questions on the distribution of primes can be cast as solving systems of linear equations with prime variables. The twin prime conjecture, Goldbach's conjecture, k-term arithmetic progressions of primes and most questions about small gaps between primes can all be seen in this manner, as well as several questions with applications to diophantine geometry or cryptography.
We will describe some of the progress on these questions, with a particular emphasis on establishing weak forms of some of these questions which has led to new results on bounded gaps between primes and large gaps between primes, amongst other things.

Talk 2: Prime values of polynomials
It is believed that any irreducible integer polynomial with no fixed prime divisor and positive lead coefficient should represent infinitely many prime numbers. This is well beyond current techniques, but Friedlander,Iwaniec and Heath-Brown have shown the multivariate polynomials $X^2 + Y^4$ and $X^3 + 2Y^3$ represent infinitely many primes, despite representing a thin set of integers. We will describe recent work showing the existence of many multivariate polynomials which each represent infinitely many primes but only a thin set of integers.

Talk 3: Primes with restricted digits
Numbers for which one digit does not occur in their digital expansion are rare - there are $O(x^{1-\epsilon})$ such integers less than x. We will talk about recent work showing that there are infinitely many prime numbers where one digit does not occur.

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Kamel Mazhouda, Universite de Monastir, Tunisia

Relations equivalent to the generalized Riemann hypothesis for a class of L-functions

In this talk, we prove that the Generalized Riemann Hypothesis (GRH) for functions in the class $\mathcal{S}^{\sharp\flat}$ containing the Selberg class is equivalent to a certain integral expression of the real part of the generalized Li coefficient $\lambda_F(n)$ associated to $F \in \mathcal{S}^{\sharp\flat}$, for positive integers n. Moreover, we deduce that the GRH is equivalent to a certain expression of $\mathrm{Re}(\lambda_F(n))$ in terms of the sum of the Chebyshev polynomials of the first kind. Then, we partially evaluate the integral expression and deduce further relations equivalent to the GRH involving the generalized Euler-Stieltjes constants of the second kind associated to F. This is joint work with Lejla Smajlović, University of Sarajevo.

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Richard McIntosh, University of Regina

A relation between the universal mock theta function g_2 and Zwegers' mu-function

In this talk I will use the modern notation $a = e^{2\pi iu}$, $b = e^{2\pi iv}$ and $q = e^{2\pi i\tau}$, where u and v are called *elliptic variables* and τ is called the *modular variable.* S.-Y. Kang proved that

$$iag_2(a,q) = \frac{\eta^4(2\tau)}{\eta^2(\tau)\vartheta(2u\,;2\tau)} + aq^{-1/4}\mu(2u,\tau\,;2\tau)\,,$$

where the Gordon-McIntosh universal mock theta function g_2 is given by

$$g_2(x,q) = \frac{1}{j(q,q^2)} \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)}}{1-xq^n}$$

and Zwegers' μ-function is defined by

$$\mu(u,v\,;\tau) = \mu(a,b,q) = \frac{A_l(a,b,q)}{\vartheta(b,q)}\,.$$

The *level k Appell-Lerch* function is defined by

$$A_k(u,v\,;\tau) = A_k(a,b,q) = a^{k/2} \sum_{n=-\infty}^{\infty} \frac{(-1)^{kn} q^{kn(n+1)/2} b^n}{1-aq^n}\,.$$

I will prove that the ϑ-quotient can be removed from Kang's identity to obtain

$$g_2(a,q) = -iq^{-1/4}\mu(u,\tau-u\,;2\tau)\,.$$

Generalizations to higher level Appell-Lerch functions will be discussed.

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Jim McLaughlin, West Chester University of Pennsylvania

Mock Theta Function Identities Deriving from Bilateral Basic Hypergeometric Series

The bilateral series corresponding to many of the third-, fifth-, sixth- and eighth order mock theta functions may be derived as special cases of general bilateral series deriving in turn from the series

$$\sum_{n=-\infty}^{\infty} \frac{(a,c;q)_n}{(b,d;q)_n} z^n.$$

Three transformation formulae for this series due to Bailey are used to derive various transformation and summation formulae for both these mock theta functions and the corresponding bilateral series. New and existing summation formulae for these bilateral series are also used to make explicit in a number of cases the fact that for a mock theta function, say $\chi(q)$, and a root of unity in a certain class, say ζ, that there is a theta function $\theta_\chi(q)$ such that

$$\lim_{q\to\zeta}(\chi(q) - \theta_\chi(q))$$

exists, as $q \to \zeta$ from within the unit circle.

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Stephen C Milne, Ohio State University

A nonterminating q-Dougall summation theorem for hypergeometric series in $U(n)$

In this talk we extend important classical one-variable summations and transformations of Bailey to multiple basic hypergeometric series very-well-poised on unitary groups $U(n+1)$. In particular, we derive multivariable generalizations of Bailey's 3-term transformation formula for ${}_8\phi_7$ series, and Bailey's nonterminating q-Dougall summation formula. As pointed out by Michael Schlosser, our nonterminating $U(n+1)$ q-Dougall summation formula yields a natural multivariable extension of Jacobi's classical identity for eighth powers of theta functions. All of this work is a consequence of the nonterminating $U(n+1)$ q-Whipple transformation formula of Milne and Newcomb. This work is joint with Sheldon L. Degenhardt

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Hugh Montgomery, University of Michigan

Littlewood polynomials

A *Littlewood polynomial* is a trigonometric polynomial whose coefficients c_n are ± 1 for n in an interval of length N, and are otherwise 0. Such trigonometric polynomials are of interest to analysts, and in signal processing. We survey what is known, what is new, and open questions regarding these intriguing functions, with special attention to the Shapiro polynomials.

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Michael Mossinghoff, Davidson College

Oscillations in sums involving the Liouville function

The Liouville function $\lambda(n)$ is the completely multiplicative arithmetic function defined by $\lambda(p) = -1$ for each prime p. Pólya investigated its summatory function $L(x) = \sum_{n \le x} \lambda(n)$, and showed for instance that the Riemann hypothesis would follow if $L(x)$ never changed sign for large x. While it has been known since the work of Haselgrove in 1958 that $L(x)$ changes sign infinitely often, the behavior of $L(x)$ and related functions remain of interest in analytic number theory. For example, some of K. Alladi's early work concerns some restricted sums of the closely related Möbius function. We describe some recent work that establishes new bounds on the magnitude of the oscillations of $L(x)$ and its weighted relatives, $L_\alpha(x) = \sum_{n \le x} \lambda(n)/n^\alpha$, where $0 \le \alpha \le 1$. This is joint work with T. Trudgian.

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Mel Nathanson, Lehman College, CUNY

Sums of sets of lattice points

If A is a nonempty subset of an additive abelian group G, then the *h-fold sumset* is

$$hA = \{x_1 + \cdots + x_h : x_i \in A_i \text{ for } i = 1, 2, \ldots, h\}.$$

We do not assume that A contains the identity, nor that A is symmetric, nor that A is finite. The set A is an (r, ℓ)*-approximate group in* G if there exists a subset X of G such that $|X| \leq \ell$ and $rA \subseteq XA$. The set A is an *asymptotic* (r, ℓ)*-approximate group* if the sumset hA is an (r, ℓ)-approximate group for all sufficiently large h. It is proved that every polytope in a real vector space is an asymptotic (r, ℓ)-approximate group, that every finite set of lattice points is an asymptotic (r, ℓ)-approximate group, and that every finite subset of every abelian group is an asymptotic (r, ℓ)-approximate group.

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Ken Ono, Emory Unversity

Riemann Hypothesis for Period Polynomials of Modular Forms

The period polynomial $r_f(z)$ for an even weight $k \geq 4$ newform $f \in S_k(\Gamma_0(N))$ is the generating function for the critical values of $L(f, s)$. It has a functional equation relating $r_f(z)$ to $r_f\left(-\frac{1}{Nz}\right)$. We prove the Riemann Hypothesis for these polynomials: that the zeros of $r_f(z)$ lie on the circle $|z| = \frac{1}{\sqrt{N}}$. We prove that these zeros are equidistributed when either k or N is large. This is joint work with Seokho Jin, Wenjun Ma, and Kannan Soundararajan.

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Frank Patane, Samford University

An Identity Connecting Theta Series Associated with Binary Quadratic Forms of Discriminant Δ *and* $\Delta\, p^2$

We give an identity which connects the theta series associated to a single binary quadratic form of discriminant Δ, to a theta series associated to a subset of binary quadratic forms of discriminant $\Delta\, p^2$. We then give an illustrative example to show how one can use this identity to derive a Lambert series decomposition in certain cases.

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Peter Paule, RISC, J. Kepler University, Linz

Alladi, Goellnitz-Gordon, and Computer Algebra

As a by-product of his recent work on partitions with non-repeating odd parts, Krishna Alladi derived an elegant modular relation in a natural combinatorial setting. Analytically this relation states the equality of an eta-quotient with $G(-q^2) - qH(-q^2)$, where $G(q)$ and $H(q)$ are the functions describing the Goellnitz-Gordon partitions. Taking this identity as a starting point, the talk describes recent algorithmic developments in connection with modular functions. A major general tool emerged from Radu's work on his Ramanujan-Kolberg package, namely a computer algebra algorithm to compute suitable presentations of subalgebras. Applications related to modular functions concern computer-assisted proving and discovery of q-series and q-product identities. Finally, along with a new proof of Alladi's modular relation, further possible algorithmic developments are discussed. The material of this talk arose in joint work with Silviu Radu.

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Janos Pintz, MTA Rényi Institute, Budapest, Hungary

On some old conjectures of Paul Erdős on the difference of consecutive primes

Over the last 90 years lot of efforts were made by excellent mathematicians (Hardy, Littlewood, Rankin, Erdős, Bombieri, Davenport and others) to understand the behavior of gaps between consecutive primes. The best result was still in 2004 the assertion that there are infinitely many prime gaps which are smaller than the respective average $\log x$ by a factor $0.248\ldots$ (Helmut Maier, 1988). Eleven years ago in joint work with Goldston and Yildirim we succeeded to show this with an arbitrarily small positive constant c in place of $0.248\ldots$. Somewhat later we showed that there are infinitely many gaps less than the square-root of the average gap size. Simultaneously we showed that under the very deep unproved Elliott-Halberstam conjecture there are infinitely many gaps of size at most 16. In a joint work with Motohashi we proved that there are infinitely many bounded gaps if the Bombieri-Vinogradov theorem can be improved somewhat for squarefree moduli. In 2013 Zhang succeeded in this way to show the existence of infinitely many bounded gaps. Eight months later simultaneously and independently Maynard and Tao showed that for an arbitrary k we have a chain of k consecutive primes in bounded intervals of length $C(k)$. While Zhang's work was able to show strong results about gaps between consecutive primes the Maynard-Tao method made able to study consecutive gaps between primes. In this lecture we present such results. To mention just one example: Erdős, Pólya and Turán conjectured 60 years ago that if we consider for an arbitrary k a linear combination (with fixed real coefficients) of k consecutive prime gaps then the expression takes infinitely many positive and negative values as well if and only if the non-zero coefficients are not all of the same sign. Some special cases were proved by Erdős (some in joint work with Turán). We prove the original conjecture using the methods of Maynard and Tao.

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Paul Pollack, University of Georgia

Phi-nomenology and torsion subgroups of CM elliptic curves

I will discuss recent results with Abbey Bourdon and Pete Clark concerning torsion subgroups of CM elliptic curves over number fields of varying degree. From the analytic point of view, the critical input comes from the theory surrounding Euler's phi-function and the "anatomical" structure of shifted primes.

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B. Ramakrishnan, Harish-Chandra Institute

Representation numbers for certain class of quadratic forms and some applications.

In this talk, we present our work on the determination of representation numbers for certain class of quadratic forms using the theory of modular forms. We make some observations by comparing similar formulas obtained by using different methods.

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Sinai Robins, Brown University and ICERM

Polyhedral Gauss sums, and polytopes with symmetry

We define certain natural finite sums of n'th roots of unity, called $G_P(n)$, that are associated to each convex integer polytope P, and which generalize the classical 1-dimensional Gauss sum $G(n)$ defined over $\mathbb{Z}/n\mathbb{Z}$, to higher dimensional abelian groups and integer polytopes. We consider the finite Weyl group $\mathcal{W}$, generated by the reflections with respect to the coordinate hyperplanes, as well as all permutations of the coordinates; further, we let $\mathcal{G}$ be the group generated by $\mathcal{W}$ as well as all integer translations in $\mathbb{Z}^d$. We prove that if P multi-tiles $\mathbb{R}^d$ under the action of $\mathcal{G}$, then we have the closed form $G_P(n) = \text{vol}(P)G(n)^d$. Conversely, we also prove that if P is a lattice tetrahedron in $\mathbb{R}^3$, of volume $1/6$, such that $G_P(n) = \text{vol}(P)G(n)^d$, for $n \in \{1, 2, 3, 4\}$, then there is an element g in $\mathcal{G}$ such that $g(P)$ is the fundamental tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, $(1, 1, 1)$.

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Larry Rolen, Pennsylvania State University

Indefinite theta functions, higher depth mock modular forms, and quantum modular forms

In this talk, I will describe several new results concerning the modularity of indefinite theta functions. From Zwegers' thesis, we know that special types of indefinite theta functions with prescribed signatures give rise to mock modular forms, which combined with important work of Andrews and others gives one road to understanding the mock theta functions of Ramanujan. Here, we will study several important examples of more general indefinite theta series inspired by physics and geometry and describe how to study the modularity properties of more complicated objects such as these, giving a glimpse into the general structure of indefinite theta functions. We will also study another class of indefinite theta functions, and we will discuss a new family of examples which give rise to quantum modular forms, and provide a family of canonical Maass waveforms whose Fourier coefficients are described by combinatorial functions with integer coefficients, placing the famous functions σ and σ^* of Andrews, Dyson, and Hickerson in a natural framework.

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Carla Savage, North Carolina State University

An Overview of Lecture Hall Partitions

Lecture hall partitions were introduced by Bousquet Mélou and Eriksson in 1997. They gained attention because of their strikingly simple generating function and their connection to Euler's partition theorem. In the time since, lecture hall partitions and their generalizations have been shown to be interesting structures with applications in combinatorics and number theory. Surprising connections have surfaced with, for example, generalizations of Euler's partition theorem, overpartitions, Göllnitz's little partition theorems, permutation statistics, Eulerian polynomials, Ehrhart theory, inversion sequences, the real-rootedness of descent polynomials of finite Coxeter groups, multiset permutations, Gorenstein cones and self-reciprocal polynomials, the restricted Eulerian polynomials of Chung and Graham, integer partitions with even indexed parts even, and pattern avoidance in permutations. We give an overview of these connections and mention some recent results.

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Michael Schlosser, University of Vienna

Elliptic extension of rook theory

Utilizing elliptic weights, we construct an elliptic analogue of rook numbers for Ferrers boards. Our elliptic rook numbers generalize Garsia and Remmel's q-rook numbers by two additional independent parameters a and b, and a nome p. These are shown to satisfy an elliptic extension of a factorization theorem which in the classical case was established by Goldman, Joichi and White and later was extended to the q-case by Garsia and Remmel. We obtain similar results for our elliptic analogues of Garsia and Remmel's q-file numbers for skyline boards. We further provide elliptic extensions of the j-attacking model introduced by Remmel and Wachs, and of Haglund and Remmel's rook theory for matchings of graphs. We actually give an extension of the latter which already generalizes the classical, non-elliptic case. Various applications of our results include elliptic analogues of (generalized) Stirling numbers of the first and second kind, Lah numbers, Abel numbers, r-restricted versions of all these, and closed form elliptic enumerations of (perfect and maximal) matchings of (complete) graphs. This is joint work with Meesue Yoo.

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Robert P. Schneider, Emory University

Arithmetic of partitions

We present a natural multiplicative theory of integer partitions (which are, of course, usually considered in terms of addition), and find many theorems of classical number theory—as well as new partition-theoretic identities—arise as particular cases of extremely general combinatorial structure laws.

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James Sellers, Pennsylvania State University

Infinitely Many Congruences Modulo 5 for 4-Colored Frobenius Partitions

In his 1984 AMS Memoir, G. E. Andrews introduced the family of functions $c\phi_k(n)$, which denotes the number of generalized Frobenius partitions of n into k colors. Recently, Baruah and Sarmah, Lin, Sellers, and Xia established several Ramanujan-like congruences for $c\phi_4(n)$ relative to different moduli. In this paper, which is joint work with Michael D. Hirschhorn (UNSW), we employ classical results in q-series, the well-known theta functions of Ramanujan, and elementary generating function manipulations to prove a characterization of $c\phi_4(10n+1)$ modulo 5 which leads to an infinite set of Ramanujan-like congruences modulo 5 satisfied by $c\phi_4$. This work greatly extends the recent work of Xia on $c\phi_4$ modulo 5.

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Andrew Sills, Georgia Southern University

A classical q-hypergeometric approach to the $A_2^{(2)}$ standard modules

In the early 1980's J. Lepowsky and R. Wilson gave the first Lie-theoretic proof of the Rogers-Ramanujan identities, and showed that they corresponded to the two inequivalent level 3 standard modules for the affine Kac-Moody Lie algebra $A_1^{(1)}$. Later, they showed that in fact the Andrews-Gordon-Bressoud generalizations of the Rogers-Ramanujan identities "explained" all the standard modules of all of $A_1^{(1)}$. The next logical step was to similarly try to explain $A_2^{(2)}$. This has proved to be much more difficult. The level 2 modules correspond to a dilated version of the two Rogers-Ramanujan identities. In his 1988 Ph.D. thesis, Stefano Capparelli discovered a pair of new partition identities via his analysis of the level 3 standard modules. Further progress in this direction stalled until Debajyoti Nandi, in his 2014 PhD thesis, found the analogous set of three new (and very complicated) Rogers-Ramanujan type partition identities corresponding to the three inequivalent level 4 standard modules. The partition theoretic explanation of higher levels of $A_2^{(2)}$ continue to elude us. The history of partition identities are inexorably linked with that of q-series/q-product identities. In this talk, we examine a classical approach to q-hypergeometric identities that appear to be associated with the $A_2^{(2)}$ standard modules as a whole, with a particular emphasis on those from levels 3 through 9.

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Peter Sin, University of Florida

Title: The critical groups of Paley graphs

To each connected graph there is associated a certain abelian group called its critical group (or sandpile group, or graph jacobian), whose order is the number of spanning trees, and which is connected to the abelian sandpile model in physics and to chip-firing games. The Paley graphs are a well-known family of graphs defined using quadratic residues in certain finite fields. We compute the critical graphs of the Paley graphs and related graphs, making use discrete Fourier transforms and properties of Jacobi sums. This is joint work with David Chandler and Qing Xiang.

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Jaebum Sohn, Yonsei University, Seoul

Hyper m-ary partition and hyper m-ary overpartition trees

In this talk, we introduce how to construct a hyper m-ary partition tree and a hyper m-ary overpartition tree. By analyzing the trees, we derive some new results.

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Kannan Soundararajan, Stanford University

Distribution of consecutive primes in residue classes

I will discuss surprising irregularities in the behavior of consecutive primes in residue classes. In joint work with Robert Lemke Oliver, we study these irregularities and formulate conjectures to explain them.

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Richard Stanley, University of Miami

Smith Normal Form and Combinatorics

Let R be a commutative ring (with identity) and A an $n \times n$ matrix over R. Suppose there exist $n \times n$ matrices P, Q invertible over R for which PAQ is a diagonal matrix $\mathrm{diag}(\alpha_1, \dots, \alpha_r, 0, \dots, 0)$, where α_i divides α_{i+1} in R. We then call PAQ a *Smith normal form* (SNF) of A. If R is a PID then an SNF always exists and is unique up to multiplication by units. We will survey some connections between SNF and combinatorics. Topics will include (1) the general theory of SNF, (2) a close connection between SNF and chip firing in graphs, (3) the SNF of a random matrix of integers (joint work with Yinghui Wang), (4) SNF of special classes of matrices, including some arising in the theory of symmetric functions and the theory of hyperplane arrangements, and (5) an example of SNF over a non-PID (with Christine Bessenrodt).

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Cameron Stewart, University of Waterloo

On the abc conjecture

We shall discuss the abc conjecture and some of the approaches that have been taken to resolve it.

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Armin Straub, University of South Alabama

Core partitions into distinct parts and an analog of Euler's theorem

A special case of an elegant result due to Anderson proves that the number of $(s, s+1)$-core partitions is finite and is given by the Catalan number C_s. Amdeberhan recently conjectured that the number of $(s, s+1)$-core partitions into distinct parts equals the Fibonacci number F_{s+1}. We prove this conjecture by enumerating, more generally, $(s, ds-1)$-core partitions into distinct parts. As a by-product of our discussion, we obtain a bijection between partitions into distinct parts and partitions into odd parts, which preserves the perimeter (that is, the largest part plus the number of parts minus 1). This simple but curious analog of Euler's theorem appears to be missing from the literature on partitions.

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Sergei Suslov, Arizona State University

Symmetry in Physics: a Way from Poincare Group Representations to Relativistic Wave Equations

We analyze kinematics of the fundamental relativistic wave equations, in a traditional way, from the viewpoint of the representation theory of the Poincaré group. In particular, the importance of the Pauli-Lubański pseudo-vector is emphasized here not only for the covariant definition of spin and helicity of a given field but also for the derivation of the corresponding equation of motion. In this consistent group-theoretical approach, the resulting wave equations occur, in general, in certain overdetermined forms, which can be reduced to the standard ones by a matrix version of Gaussian elimination. Although, mathematically, all representations of the Poincaré group are locally equivalent, their explicit realizations in conventional linear spaces of four-vectors and tensors, spinors and bispinors, etc. are quite different from the viewpoint of physics. This is why, the corresponding relativistic wave equations are so different. Among them we concentrate on Dirac's equations, Weyl's two-component equation for massless neutrinos, and the Proca, Maxwell, and Fierz-Pauli equations. The case of linearized Einstein's equations for weak gravitational fields is also discussed.

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Gérald Tenenbaum, Université de Lorraine

Effective mean-value estimates for complex multiplicative functions

New, effective mean-value estimates for a fairly wide class of complex multiplicative arithmetic functions will be described. These provide (essentially optimal) quantitative versions of Wirsing's classical estimates and extend those of Halász. Several applications will be briefly presented, including: estimates for the difference of mean-values of so-called pretentious fonctions; local laws for the distribution of prime factors in an arbitrary set, and weighted distribution of additive functions.

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Sarah Trebat-Leder, Emory University

A new moonshine for the Mathieu group M_{23}

We show that for the Matthieu group M_{23}, there exists an infinite-dimensional graded module M such that the graded trace of $g \in M_{23}$ acting on M is, up to the constant term, identical to a monstrous moonshine McKay-Thompson series. This new moonshine comes from Matthieu moonshine, but is significantly simpler, as the functions involved are modular of weight 0 on genus zero subgroups instead of mock modular of weight 1/2.

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Alexandre Turull, University of Florida

Measures of the complexity of finite groups and their bounds

There are many ways to measure the complexity of a finite group. We will focus on only a few of them. We will start with a famous Theorem of John Thompson, and some of the measures and theorems that it eventually suggested down the road. We will discuss some related results involving number theory, including a joint paper of the speaker with Krishnaswami Alladi and Ron Solomon. In addition, we will discuss a different type of bound, originating by a famous Theorem of Jordan. We will discuss some related more recent work of the speaker and Ignasi Mundet i Riera giving abstract tools to prove the existence of such bounds.

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Ali Uncu, University of Florida

Some Weighted Partition Identities and Dyson Crank

We provide a different angle to Alladi's weighted partition identity relating Rogers-Ramanujan type partitions with unrestricted partitions. We then present weighted partition identities connecting some of the new weights with the Dyson crank.

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Robert Vaughan, Pennsylvania State University

The asymptotic formula in Waring's problem: Higher order expansions

When $k > 1$ and s is sufficiently large in terms of k, we derive an explicit multi-term asymptotic expansion for the number of representations of a large natural number as the sum of s positive integral kth powers.

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Christophe Vignat, Tulane University

Narayana polynomials and random walks in space

The Narayana numbers and polynomials, and their connection with moments of Pearson random walks in space will be described. A generalization of the Narayana polynomials will be given and related to the classical Gegenbauer polynomials: the probabilistic interpretation of a sequence related to the Narayana numbers will be used to prove some results first introduced by M. Lassalle. This is joint work with T. Amdeberhan, V.H. Moll, J. Borwein and A. Straub.

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Ole Warnaar, The University of Queensland, Australia

Virtual Koornwinder integrals

In this talk, based on joint work with Eric Rains, I will explain how Virtual Koornwinder integrals may be used to prove Rogers-Ramanujan identities for affine Lie algebras.

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Michael Woodbury, University of Cologne

Generalized Frobenius partitions and powers of the Jacobi theta function

Let $\vartheta(z;\tau) := \sum_{n\in\mathbb{Z}} e^{\pi i n^2\tau + 2\pi i n(z+\frac{1}{2})}$ be the Jacobi theta function. If we let $q := e^{2\pi i\tau}$ and $\zeta := e^{2\pi i z}$, then we are interested in computing the q-series $F_{k,n}(q)$ in the Fourier expansion $\theta(z;\tau)^k = \sum_{n\in\mathbb{Q}} F_{k,n}(q)\zeta^n$. We give a general recursive formula for finding the q-series $F_{k,n}(q)$. Of particular interest is $F_{k,\frac{k}{2}}(q)$ because of its connection to certain generalized Frobenius partitions. For example, when $k = 1$, up to some normalization factor, $F_{1,\frac{1}{2}}(q)$ is the generating series for the partition function. In the talk, our recursive formula and its connection to generalized Frobenius partitions will be discussed. This is joint work with Kathrin Bringmann and Larry Rolen.

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Ae Ja Yee, Pennsylvania State University

Legendre Theorems for subclasses of partitions/overpartitions

A. M. Legendre noted that Euler's pentagonal number theorem implies that the number of partitions of n into an even number of distinct parts almost always equals the number of partitions of n into an odd number of distinct parts (the exceptions occur when n is a pentagonal number). Subsequently other classes of partitions, including overpartitions, have yielded related Legendre theorems. In this talk, we examine some subclasses of partitions and overpartitions that have surprising Legendre theorems. This is joint work with George Andrews.

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Doron Zeilberger, Rutgers University

Krishna Alladi's Early Love: Irrationality Measures of Famous Constants

In a beautiful paper, joint with M.L. Robinson, "Legendre polynomials and irrationality", Krishna Alladi adapted Beukers' method to find irrationality measures of very famous constants. I will survey this, and also describe attempts to teach my silicon collaborator, Shalosh B. Ekhad, to use Krishna's ideas.

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Wadim Zudilin, University of Newcastle, Australia

On certain irrational values of the logarithm: beyond the 1979 limitations

In a note "On certain irrational values of the logarithm" by Alladi and Robinson, a family of integrals was introduced to investigate the irrationality of logarithms of some (simple) algebraic numbers. (The integrals were inspired by Beukers' proof of Apéry's theorem about the irrationality of $\zeta(2)$ and $\zeta(3)$.) The note concluded with a discussion of natural limitations of the method, for example, its failure to approach the irrationality of $\log(3)$ and π. In my talk I will explain how the Alladi–Robinson integrals, without any modification, are used in establishing that the latter two numbers are irrational.

Title of Special Colloquium: Short random walks and Mahler measures

An n-step uniform random walk is a walk that starts at the origin and consists of n steps of length 1 each taken into a uniformly random direction. It is particularly interesting for $n = 2, 3, 4, 5$ because of its beautiful links to modular and hypergeometric functions. The Mahler measure of an n-variable polynomial is its geometric mean over the n-dimensional torus. There are several cases when n-variate Mahler measures are known or conjectured to be linked to hypergeometric functions and noncritical L-values, for small n as well. In the talk I will outline the links above and indicate some new interconnections between the short random walks and Mahler measures. These novel results are from joint work in progress with Armin Straub.

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