Preface

We are extremely pleased to offer these proceedings of the Gainesville Number Theory Conference of 2016, more affectionately known as ALLADI60, honoring Krishna Alladi on his 60th birthday. Krishna has been a major contributor to number theory and mathematics in several ways. First he is a first class mathematician. We who have collaborated with him are most vividly aware of his insight and talent. Second, he is Editor-in-chief of the Ramanujan Journal and a Series Editor of Developments in Mathematics. Third, he instituted the internationally admired SASTRA Ramanujan Prize. Fourth, he has been the major actor in the creation of numerous important conferences.

In the past two decades, the University of Florida has been the main international venue for conferences in the areas of partitions, \( q \)-series, modular forms, and Ramanujan’s work. The special feature of this conference was that in addition to these areas, analytic number theory, irrationality and transcendence were also covered — areas that Alladi had worked on until 1990.

The conference attracted nearly 200 participants and was spread over five days to accommodate the nearly 100 speakers who had come from Australia, Austria, Canada, China, England, France, Germany, Hong Kong, Hungary, India, Israel, Korea, The Netherlands, New Zealand, Norway, Serbia, Switzerland, Tunisia, Turkey, and the United States. The conference was supported by grants from the National Science Foundation, the National Security Agency, the Number Theory Foundation and by funds from The Pennsylvania State University. Local support was provided by the University of Florida, Mathematics Department and the College of Liberal Arts and Sciences, and the Alachua County Tourism Board. We are most grateful for all this support which helped make the conference a success.

**Special Lectures**

There were four special lectures at the conference:
Opening Lecture
Manjul Bhargava (Princeton University),
*Squarefree values of polynomial discriminants*
In 2003, Manjul Bhargava won the First SASTRA Ramanujan Prize, a prize that Krishna Alladi was instrumental in launching.

The Erdős Colloquium
Hugh L. Montgomery (University of Michigan)
*Littlewood polynomials*
This Special Colloquium, initiated by Krishna Alladi, has been given yearly at the University of Florida since 1999. Following the Colloquium, the participants were treated to a dinner party at the home of Krishna and Mathura Alladi, and our thanks to Mathura for graciously hosting this.

The Ramanujan Colloquium
James Maynard (University of Oxford)
*Linear equations in primes*
This Special Colloquium, initiated by Krishna Alladi, has been given yearly at the University of Florida since 2007, and is sponsored by The Pennsylvania State University and George Andrews. James Maynard also gave two other lectures.

Conference Closing Lecture and Math Colloquium
Wadim Zudilin (University of Newcastle)
*Short random walks and Mahler measures*
Wadim Zudilin also gave a lecture on certain irrational values of the logarithm, which was related to Krishna Alladi’s work from 1979.

Other Conference Highlights

Piano Recital
Following Maynard’s Colloquium, there was a Reception at the Keene Faculty Center where the participants were treated to a lovely piano concert by Christian Krattenthaler of the University of Vienna.

Awards
Ron Graham, former President of the AMS, presented cheques to James Maynard and Kevin Ford in recognition of their work for the resolution of a famous $10,000 problem of Erdős on large gaps between primes. Maynard received $5,000 for his solo paper, and Kevin Ford received a cheque for $5000 made out to Kevin Ford, Ben Green, S. Konyagin and Terence Tao for their joint work.

Soundararajan’s Lecture
Another SASTRA Ramanujan prize winner, Kannan Soundararajan (Stanford), presented for the first time his recent work with his colleague Robert Lemke Olivera on a startling new result on a bias in the distribution of consecutive prime numbers.

The Man Who Knew Infinity
The movie *The Man Who Knew Infinity* on the remarkable life of Ramanujan was shown at the conference as a Preview before its official opening in theaters thanks to the efforts of Manjul Bhargava, one of the Associate Producers of the movie. Our thanks to Edward Pressman films for this kind gesture. See Bruce Berndt’s paper in this volume for some background on an interesting scene from the movie.

**Talks on Alladi’s Work**

Several talks at the conference dealt with Alladi’s work – not just his recent work on partitions and $q$-series, but his early work as well. Dorian Goldfeld (Columbia University) spoke about extensions of results in Alladi’s first paper (written when he was an undergraduate) with Paul Erdős on an additive arithmetic function – now called the Alladi-Erdős function. Doron Zeilberger (Rutgers University) and Wadim Zudilin (University of Newcastle, Australia) discussed extensions of results of Alladi-Robinson (1980) on irrationality measures.

**Conference Banquet**

A Conference Banquet was held at the Paramount Hotel. Many speeches honoring Krishna were given. The text of Elizabeth Loew’s speech is given after this preface.

**Mathematical Interests of Krishna Alladi**

Over his career Krishna Alladi has maintained an interest in Number Theory, Combinatorics, Discrete Mathematics, Analytic Number Theory, Sieve Methods, Probabilistic Number Theory, Diophantine Approximations, Partitions, and $q$-Series Identities. His research in mathematics began as an 18 year old undergraduate in 1973. The first part of his mathematical career was in Analytic Number Theory. In particular, he wrote 5 joint papers with Paul Erdős. In 1987, the Ramanujan Centenary year, Krishna became interested in partitions and in the early 1990s he began a fruitful collaboration with George Andrews, Alex Berkovich, and Basil Gordon, when they made impressive breakthroughs in discovering partition identities beyond those of Rogers-Ramanujan, Schur and Göllnitz. In 1993, he spent a sabbatical at Penn State with George Andrews. There he learned the importance of basic hypergeometric series, and modular forms for the theory of partitions.

**Volume Contents**

Below we give a brief description of the papers in this volume and group them according to these topics: Analytic Number Theory, Probabilistic Number Theory, Partitions, Basic Hypergeometric Functions, and Modular Forms.
Analytic Number Theory

Benli, Elma and Yidirim extend Conrey and Ghosh’s results on zeros of derivatives of the Riemann zeta-function near the critical line, to Dirichlet $L$-functions.

Deshouillers and Grekos study the problem of the number of integral points on a convex curve in terms of length and curvature, and make improvements on previous results.

In 1977 Alladi and Erdős showed that a certain important additive function is uniformly distributed modulo 2. Goldfeld generalizes this result to an arbitrary modulus.

Montgomery’s survey paper on Littlewood Polynomials is an expanded version of his talk given at the conference.

Nicolas obtains an effective version of Ramanujan’s result for the difference between the logarithmic integral of Chebychev’s function and $\pi(x)$.

Ono, Schneider and Wagner prove an partition theoretic analog of Alladi’s Möbius function identity.

Saradha and Sharma prove some conjectures of Mueller and Schmidt for the number of integer solutions of a Thue inequality for certain binary quadratic forms.

Tenenbaum extends a result of Mertens for the sum of the reciprocals of primes to the sum of the reciprocals of the product of $k$ primes.

Inspired by Hugh Montgomery’s talk on Littlewood polynomial, Zeilberger (with his computer collaborator Ekhad) gives algorithmic approach to Saffari’s conjecture on the asymptotic growth of moments of the Rudin-Shapiro polynomials.

Probabilistic Number Theory

Elliott’s paper is survey of abstract multiplicative functions and their application to the study of the Fourier coefficients of automorphic forms, together with a discussion in the context of the theory of Probabilistic Number Theory.

Partitions

George Andrews recently gave a refinement of Krishna Alladi’s variant of Schur’s 1926 partition theorem. In his paper, Andrews develops a surprising factorization of the related polynomial generating functions.

Chen, Ji, and Zang previously proved a rank-crank inequality conjecture of Andrews, Dyson and Rhoades. By using combinatorial methods they show that there is a reordering of partitions that explains the very nearly equal distributions of the rank and the crank.

Motivated by recent research of Krishna Alladi, Berkovich and Uncu give new weighted partition identities for partitions, overpartitions, and partitions with distinct even parts, using the theory of basic hypergeometric functions.
Dousse extends Krishna Alladi’s method of weighted words to obtain generalizations and refinements for previous extensions of Schur’s partition theorem to overpartitions due to Andrews, Corteel, and Lovejoy.

Kolitsch gives new partition interpretations of truncated forms of Euler’s Pentagonal Number Theorem and Jacobi’s Triple Product identity in terms of overpartitions.

Krattenthaler finds congruences mod 16 for the number of unique path partitions of \( n \), which occur in the study of character values of finite symmetric groups, and which generalize results of Olsson, Bessendrodt, and Sellers.

Kanade, Kurşungöz and Russel give combinatorial interpretations of overpartition variants of Andrews’s \( H \) and \( J \) functions which occurred in the study of the Andrews-Gordon partition identities and their generalizations.

Lovejoy gives two overpartition extensions of Alladi and Gordons generalization of Schurs theorem.

Bringmann and Mahlburg present new companions to the Capparelli partition identities and two new general identities for three-color partitions that may be specialized to theta functions and false theta functions.

Seo and Yee give a combinatorial proof of a result of Andrews which is an overpartition analog of Rogers-Ramanujan type theorem related to restricted successive ranks.

**q-Series and Basic Hypergeometric Functions**

Gaurav Bhatnagar gives a marvelous bibasic version of Heine’s basic hypergeometric transformation and uses it to prove and organize a raft of identities of Ramanujan, some of which are easy and some which are not.

Cooper, Wan, and Zudilin prove a number of Z.-W. Sun’s conjectures for series for \( 1/\pi \) by relating them to known series using techniques of basic hypergeometric series and Zeilberger’s algorithm for holonomic sequences.

Banerjee and Dixit obtain new identities for Ramanujan’s function \( \sigma(q) \) which is the generating function for the excess number of partitions of \( n \) into distinct parts with even rank over those of odd rank.

Hirschhorn gives elementary proofs of some well known arithmetic properties of Ramanujan’s tau function using nothing more than Jacobi’s triple product identity.

Liu describes a method for finding certain series expansions of functions that satisfy a \( q \)-partial differential equation and, as an application, finds a generalization of Andrews’s transformation formula for the \( q \)-Lauricella function.

Mc Laughlin gives a new approach using bilateral hypergeometric series to obtain identities for mock theta functions and finds radial limit formulas as an application.

Schlosser and Yoo employ a one-variable extension of \( q \)-rook theory to give combinatorial proofs of some basic hypergeometric summations formulas.
Sills gives an elementary approach for finding the sum side of Rogers-Ramanujan type identities from the product forms related to the standard modules of the Kac-Moody algebra $A_2^{(2)}$.

**Modular Forms**

Nicolas Andersen follows up on his previous work, "Vector-Valued Modular Forms and The Mock Theta Conjectures," where he gave a new proof of Ramanujan’s fifth order mock theta conjectures using the theory of vector-valued modular forms and harmonic Maass forms. In his new paper he extends these ideas to give a new proof of Hickerson’s seventh order identities.

Jha and Kumar compute the adjoint (with respect to the Petersson inner product) of the linear map related to the Cohen-Rankin bracket, thus extending work of Kohnen and Herrero to half-integral weight modular forms.

Kimport obtains asymptotic expansions for weight $1/2$ and $3/2$ partial theta functions at roots of unity which generalizes results of Berndt and Kim and are important in the study of certain quantum modular forms.

McIntosh proves that Zweger’s $\mu$-function, which is important in the study of mock theta functions from a modular form view, is essentially no more general than the universal mock theta function $g_2$.

Paule and Radu derive a new type of modular function identity that implies Ramanujan’s partition congruence mod 11.

Ramakrishnan, Sahu, and Singh use the theory of modular forms to find formulas for the number of representations of a positive integer by certain class of quadratic forms in eight variables.

**Jon Borwein**

Jon Borwein was unable to come to ALLADI60 due to his commitment to give a series of lectures as a Distinguished Scholar in Residence at Western University, London Ontario. Later in May, 2016, Jon and his wife Judi were able to visit us in Florida. He gave two talks — one at the University of Florida Brain Institute on CARMA: A Model for Multi-Discipline and Multi-Institution Collaboration. The other talk on Seeing Things by Walking on Numbers was given in the Math Department. Later in July, I (Frank) was with Jon at the Lambert Conference in London, Ontario. It was a great shock to us that he died just a few days after the Lambert Conference, and we still feel the loss. Jon was the first person to submit a paper to our proceedings. The referee only required some minor revisions. Jon wanted to wait until he got back to Australia to complete these revisions but sadly this did not happen. I made the revisions myself and got David Bailey to check them over. Jon’s paper is an expanded companion to a talk he gave at a workshop celebrating Tony Guttmann’s 70th birthday. It describes his encounters over nearly 30 years with
Sloane’s (Online) Encyclopedia of Integer Sequences. We agree with the referee that it is a masterpiece, with beautiful math and beautiful exposition.

**Alladi Ramakrishnan**

Krishna’s father, the late Professor Alladi Ramakrishnan, the Founder-Director of MATSCIENCE, The Institute of Mathematical Sciences in Madras, India, was an inspiration to Krishna and supported all of Krishna’s efforts. In an emotionally charged speech at the banquet, Krishna said that if his father were alive, he would have been the happiest person to see such an impressive gathering of mathematicians from around the world for the 60th birthday conference. Several speakers in their speeches at the banquet made references to Krishna’s father. Kryuchkov, Lankfear, and Suslov have dedicated their paper to the memory of Krishna’s father, the famous physicist Professor Alladi Ramakrishnan, on the topic of a complex form of classical and quantum electrodynamics.

**Thanks**

We thank Marc Strauss and Elizabeth Loew for the Springer book exhibit and Rochelle Kronzek for the World Scientific book exhibit. In addition we thank Marc Strauss for publishing these proceedings in the Springer Proceedings Series. We express special thanks to Margaret Somers, Cyndi Garvan, Ali Uncu and Chris Jennings-Schaffer, the staff of the Math Department and the number theory graduate students for all aspects of preparing for and running a smooth conference.

We conclude by expressing again our thanks to Krishna for his monumental contributions. Our community has been greatly enriched by him, and we are deeply in his debt.

Gainesville, Florida, August, 2017

George E. Andrews
Frank Garvan