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# FOCUSED WEEK ON INTEGRAL LATTICES

February 15 – 20, 2010

University of Florida, Gainesville, FL 32611

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## ABSTRACTS

**George Andrews**, Pennsylvania State University  
4:00 – 4:55PM, MONDAY, FEBRUARY 15

*THE WORLD OF  $q$  — Orthogonal Polynomials and Bailey Chains*

This talk will introduce  $q$ -orthogonal polynomials and will begin with a brief (and I hope compelling) account of their background and history.

In 1975-76, Richard Askey and I extended the ideas of Wolfgang Hahn on orthogonal polynomials arising in the world of  $q$ . In a couple of papers, we established a variety of connection coefficient theorems from which various Rogers-Ramanujan identities followed.

In the early 1980's, the discovery of Bailey Chains and their iterative power led to a neglect of  $q$ -orthogonal polynomials. Recently the importance of their role has emerged again. The final portion of the talk will be devoted to an explanation of this re-emergence.

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**Eiichi Bannai**, Kyushu University, Japan  
12:50 – 1:40PM, TUESDAY, FEBRUARY 16

*Spherical designs and toy models for D. H. Lehmer's conjecture*

(This talk is based on joint work with Tsuyoshi Miezeki.)

In 1947, Lehmer conjectured that the Ramanujan  $\tau$ -function  $\tau(m)$  never vanishes for all positive integers  $m$ , where the  $\tau(m)$  are the Fourier coefficients of the cusp form  $\Delta_{24}$  of weight 12. Lehmer verified the conjecture in 1947 for  $m < 214928639999$ . In 1985, Serre verified the conjecture up to  $m < 10^{15}$ , and in 1999, Jordan and Kelly for  $m < 22689242781695999$ , and so on. The theory of spherical  $t$ -designs, and in particular of those which are the shells of Euclidean lattices, is closely related to the theory of modular forms, as first shown by Venkov in 1984. In particular, Ramanujan's  $\tau$ -function gives the coefficients of a weighted theta series of the  $E_8$ -lattice. It is shown, by Venkov, de la Harpe, and Pache, that  $\tau(m) = 0$  is equivalent to the fact that the shell of norm  $2m$  of the  $E_8$ -lattice is an 8-design. So, Lehmer's conjecture is reformulated in terms of spherical  $t$ -design. Lehmer's conjecture is difficult to prove, and still remains open. In this talk, we consider toy models of Lehmer's conjecture. Namely, we show that the  $m$ -th Fourier coefficient of the weighted theta series of the  $\mathbb{Z}^2$ -lattice and the  $A_2$ -lattice does not vanish, when the shell of norm  $m$  of those lattices is not the empty set. In other words, the spherical 4 (resp. 6)-design does not exist among the shells in the  $\mathbb{Z}^2$ -lattice (resp.  $A_2$ -lattice). Further toy models for certain two dimensional lattices will also be mentioned. Also, I will discuss our recent joint work with Vladimir Yudin on more elementary (i.e., modular form free) approach to these toy models.

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**David Bressoud**, Macalester College  
4:00 – 4:40PM, THURSDAY, FEBRUARY 18

*Issues of the Transition to College Mathematics*

Over the past quarter century, 2- and 4-year college enrollment in first semester calculus has remained constant while high school enrollment in calculus has grown tenfold, from 60,000 to 600,000, and continues to grow at 6% per year. We have passed the crossover point where each year more students study first semester calculus in US high schools than in all 2- and 4-year colleges and universities in the United States. In theory, this should be an engine for directing more students toward careers in science, engineering, and mathematics. In fact, it is having the opposite effect. This talk will present what is known about the effects of this growth and what needs to happen in response within our high schools and universities.

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**Wai Kiu Chan**, Wesleyan University  
4:05 – 4:55PM, TUESDAY, FEBRUARY 16

*Sums of Triangular Numbers*

In 1796 Gauss wrote in his mathematical diary the theorem that every natural number is the sum of three triangular numbers. In today's terminology, we say that the sum of three triangular numbers is universal. Later Liouville proved a generalization of Gauss' theorem which determines all ternary sums of triangular numbers that are universal. In this talk, I will describe the recent result by B. Kane which provides a simple characterization of universal sums of triangular numbers and my joint work with B.K. Oh on almost universal sums of triangular numbers.

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**Robert Griess**, Univeristy of Michigan  
10:40 – 11:30AM, THURSDAY, FEBRUARY 18

*New lattices with moderately high minimum norms*

Lately, we have been studying lattice constructions with finite group techniques.

We exhibit families of even unimodular lattices with moderately high minimum norms, namely 6 in rank 72 and 8 in ranks 96, 120 and 128. The extremal upper bounds for these respective dimensions are 8, 10, 12, 12. (The rank 72 examples could possibly have minimum norm 8, but we claim only at least 6). Variations on older techniques are shown to give our high minimum norms if we input lattices with nice isometry groups.

Other lattice constructions using groups may be sketched, if time permits.

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**Pham Tiep**, University of Arizona  
3:00 – 3:50PM, WEDNESDAY, FEBRUARY 17

*Integral lattices and rational representations*

In the first part of the talk we will survey constructions and properties of several series of integral lattices. In the second part we will discuss recent results on rational representations of finite groups.

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