FOCUSED WEEK ON QUADRATIC FORMS AND THETA FUNCTIONS

March 22 – 26, 2010

University of Florida, Gainesville, FL 32611

ABSTRACTS

George Andrews, Pennsylvania State University
4:05 - 4:55PM, FRIDAY, MARCH 26

New Aspects of False Theta Functions
Recently, I have been looking at Bailey's Lemma and Bailey pairs from the point of view of orthogonal polynomials. This perspective has yielded a number of new combinatorial results mostly involving the false theta functions of L.J. Rogers. We shall discuss the efficacy of this approach and shall apply it to various partition theoretic and combinatorial questions including "concave compositions" and "concatenable spiral self-avoiding walks."

⋆ ⋆ ⋆

Roger Baker, Bringham Young University
3:00 - 3:50PM, FRIDAY, MARCH 26

Asymptotic Formulas for Positive Definite Quadratic Forms
Let Q be a positive definite integral quadratic form in 2 or more variables. Let $r(Q, n)$ be the number of representations of n by Q (using integer vectors). One might expect to find in the literature an asymptotic formula for the sum of the squares of the $r(Q, n)$ for n up to x, but this is not so, except for special cases. For example, it has been done for only finitely many binary forms. In this talk a general solution of the problem is given. An effort to put the constant multiplier in the asymptotic formula into closed form leads to a curious problem about quadratic forms over the ring of integers modulo 4.

⋆ ⋆ ⋆

Bruce Berndt, University of Illinois
TALK 1: 3:00 - 3:50PM, MONDAY, MARCH 22

The Circle Problem, The Divisor Problem, and Two Bessel Series Expansions in Ramanujan's Lost Notebook
On page 335 in the volume containing his Lost Notebook, Ramanujan recorded two identities, each involving a double series of Bessel functions. One of the identities has been proved (in a sense), while the other is close to being proved (in a sense). One is connected with the famous *circle problem*, while the other is connected with the equally famous *divisor problem*. We review these impenatrable,

unsolved problems, discuss what we know about them, and show their connections with Ramanujan's two formulas. A discussions of proofs and attempted proofs of Ramanujan's formulas will be a focus of our expository talk.

TALK 2: 3:00 - 3:30PM, WEDNESDAY, MARCH 24

Questions about Four Sets of Problems from Ramanujan's Lost Notebook

⋆ ⋆ ⋆

Byungchan Kim, University of Illinois
1:55 - 2:45PM, THURSDAY, MARCH 25

Asymptotic Formulas for Some Theta Functions
In this talk, we will discuss asymptotic formulas for some variants of theta functions and their connection to combinatorics. The main objects of this talk will be

1. asymptotic formula for the number of t-core partitions,
2. periodicity for signs of Fourier coefficient for certain eta-quotients,
3. asymptotic expansion for partial theta functions.

⋆ ⋆ ⋆

Atul Dixit, University of llinois
9:35 - 10:25AM, TUESDAY, MARCH 23

Transformation Formulas Associated with Integrals Involving the Riemann Xi Function
Page 220 of Ramanujan's Lost Notebook contains a beautiful transformation formula involving the digamma function which is also associated with an integral involving the Riemann Xi function. Here we discuss some new transformation formulas of this type, of which one generalizes the above-mentioned formula of Ramanujan. Also included are new extensions of formulas of N.S. Koshliakov, A.P. Guinand and W.L. Ferrar. The interesting history behind some of these formulas will also be discussed. The talk will conclude with some open problems in this area.

⋆ ⋆ ⋆

William Jagy, MSRI Berkeley
12:50 - 1:40PM, TUESDAY, MARCH 23

Ternary Quadratic Forms
We present several unusual phenomena involving integral positive ternary quadratic forms. Included are excerpts from work with Alexander Berkovich giving new relations among theta functions.

⋆ ⋆ ⋆

Brandt Kronholm, SUNY at Albany
3:00 - 3:50PM, THURSDAY, MARCH 25

Generalized Ramanujan Congruence Properties of the Restricted Partition Function $p(n,m)$
The restricted partition function $p(n,m)$ enumerates the number of partitions of n into exactly m parts. The relationship between the unrestricted partition function $p(n)$ and $p(n,m)$ is clear:

$$p(n) = p(n,1) + p(n,2) + ... + p(n,n).$$

In 1919 Ramanujan observed proved the following partition congruences:

$$p(5n+4) \equiv 0 \pmod 5$$
$$p(7n+5) \equiv 0 \pmod 7$$
$$p(11n+6) \equiv 0 \pmod{11}$$

Ono (2000) proved that there are such congruences for $p(n)$ modulo every prime $\ell > 3$.
Ramanujan further conjectured a generalization for a modulus of powers of 5, 7, and 11 which was eventually proved by Atkin in 1967.
In this talk we will discuss a Ramanujan-like congruence relation for $p(n,m)$ where for our choice of any choice of prime power modulus there is no restriction on n. We will highlight a few of the results and techniques in the several papers (Kwong, Nijenhuis, Wilf) preceeding our main result.
If time permits, we will consider future research regarding $p(n,m)$.

⋆ ⋆ ⋆

Steve Milne, Ohio State University
4:05 - 4:55PM, TUESDAY, MARCH 23

Sums of Squares, Schur Functions, and Multiple basic Hypergeometric Series
We start with a brief review of the sums of squares problem from Jacobi, Glaisher, and Ramanujan, on to the present, followed by a quick summary of one-variable basic hypergeometric series. We then give some necessary background on multiple basic hypergeometric series and their connection to the sums of squares problem. Motivated by Andrews' (one-variable) basic hypergeometric series proof of Jacobi's 2, 4, 6, and 8 squares identities, we discuss how we used multiple basic hypergeometric series, Gustafson's C_ℓ nonterminating ${}_6\phi_5$ summation theorem, and symmetry and Schur function techniques to prove the existence of three basic types of infinite families of explicit exact non-trivial closed formulas for the number of ways of writing a positive integer N as a sum of a given

number of squares of integers, without using coefficients of cusp forms. These three types of infinite families involve either $4n^2$ or $4n(n+1)$ squares, $2n(2n-1)$ or $2n(2n+1)$ squares, or n^2 or $n(n+1)$ squares, respectively. The $n = 1$ case is classical. We first computed the explicit $n = 2$, and/or $n = 3$ cases by the aid of Mathematica. All of these identities, even for $n = 2$ or $n = 3$, for example 9, 12, and 20 squares, are of significant interest. We briefly note how we subsequently used combinatorial/elliptic function methods to actually derive these explicit exact non-trivial closed formulas, from our first type of infinite families, for $4n^2$ or $4n(n+1)$ squares of integers, respectively. Similar combinatorial/elliptic function methods allow us to recover the $n = 2$ case of our $2n(2n-1)$ or $2n(2n+1)$ squares identities. These 12 and 20 squares formulas, which extend Jacobi's 2 and 6 squares identities, respectively, are in the form of 2 by 2 determinants of Lambert series that expand $\vartheta_3(0, -q)$ to the 12-th and 20-th power, respectively. These are not Hankel determinants as in the $4n^2$ or $4n(n+1)$ squares identities, but a subtle variation. Our methods should also work for the general $2n(2n-1)$ or $2n(2n+1)$ squares case, yielding deep extensions of Hankel determinants. In addition to the three types of infinite families of sums of squares identities discussed above, we are also able to obtain the product sides of many infinite families of Macdonald's η-function identities.

⋆ ⋆ ⋆

Michael Schlosser, University of Vienna, Austria
4:05 - 4:55PM, MONDAY, MARCH 22

Special Commutation Relations and Combinatorial Identities
We study commutation relations involving special weight functions, for which we obtain a weight-dependent generalization of the binomial theorem. In the notable special case of the weight functions being suitably chosen elliptic (i.e., doubly-periodic meromorphic) functions, our algebra consists of, what we call, "elliptic-commuting" variables (which generalize the q-commuting variables with $yx = qxy$). These are shown to satisfy an elliptic generalization of the binomial theorem. The latter can be utilized to quickly recover Frenkel and Turaev's ${}_{10}V_9$ summation formula, an identity fundamental to the (rather young) theory of elliptic hypergeometric series. Furthermore, the combinatorial interpretation of our commutation relations in terms of weighted lattice paths allows us to deduce other combinatorial identities as well.

⋆ ⋆ ⋆

Sun Kim, University of Illinois
9:35 - 10:25AM, THURSDAY, MARCH 25

Proof of Ramanujan's Second Bessel Function Identity
On page 335 in his Lost Notebook, Ramanujan recorded two identities involving a double series of Bessel functions. The first assertion was proved by B. C. Berndt and A. Zaharescu in 2006. In this talk, we establish a proof of the second assertion. We remark that it is connected with the famous divisor problem. We also show that the same method gives another proof of the first assertion.

⋆ ⋆ ⋆

Kannan Soundararajan, Stanford University
TALK 1: 4:00 - 4:55PM, WEDNESDAY, MARCH 24

Quantum Unique Ergodicity and Number Theory
A fundamental problem in the area of quantum chaos is to understand the distribution of high eigenvalue eigenfunctions of the Laplacian on certain Riemannian manifolds. A particular case which is of interest to number theorists concerns hyperbolic manifolds arising as a quotient of the upper half-plane by a discrete "arithmetic" subgroup of $SL_2(\mathbb{R})$ (for example, $SL_2(\mathbb{Z})$, and in this case the corresponding eigenfunctions are called Maass cusp forms). In this case, Rudnick and Sarnak have conjectured that the high energy eigenfunctions become equi-distributed. I will discuss some recent progress which has led to a resolution of this conjecture, and also on a holomorphic analog for classical modular forms. I will not assume any familiarity with these topics, and the talk should be accessible to graduate students.

TALK 2: 10:40 - 11:30AM, THURSDAY, MARCH 25

Mean Values of Multiplicative Functions and Applications
The pioneering results of Wirsing and Halasz describe the situations when the mean-value of a multiplicative function can be large. Understanding the structure of such multiplicative functions has proved useful in applications to the Polya-Vinogradov inequality, weak subconvexity bounds for L-functions, and to the quantum unique ergodicity problem. I will try and explain some of these results and applications.

TALK 3: 10:40 - 11:40AM, FRIDAY, MARCH 26

The Distribution of Values of zeta and L-Functions
I will survey what is known and expected about the distribution of values of L-functions. In particular, I will try to explain probabilistic models that describe the behavior of these functions. At the center of the critical strip, the probabilistic models arise from random matrix theory, and I will discuss the Keating-Snaith conjectures for moments of L-functions, and recent progress.

⋆ ⋆ ⋆