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SYMBOLIC COMPUTATION, NUMBER THE-  
ORY, SPECIAL FUNCTIONS, PHYSICS AND  
COMBINATORICS



SYMBOLIC COMPUTATION, NUMBER THEORY,  
SPECIAL FUNCTIONS, PHYSICS AND  
COMBINATORICS

Proceedings of the conference held at the  
Department of Mathematics, University of  
Florida, Gainesville, November 11–13, 1999

Edited by

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## Preface

These are the proceedings of the conference “Symbolic Computation, Number Theory, Special Functions, Physics and Combinatorics” held at the Department of Mathematics, University of Florida, Gainesville, from November 11 to 13, 1999. The main emphasis of the conference was Computer Algebra (i.e. symbolic computation) and how it related to the fields of Number Theory, Special Functions, Physics and Combinatorics. A subject that is common to all of these fields is  $q$ -series. We brought together those who do symbolic computation with  $q$ -series and those who need  $q$ -series including workers in Physics and Combinatorics. The goal of the conference was to inform mathematicians and physicists who use  $q$ -series of the latest developments in the field of  $q$ -series and especially how symbolic computation has aided these developments.

Over 60 people were invited to participate in the conference. We ended up having 45 participants at the conference, including six one hour plenary speakers and 28 half hour speakers. There were talks in all the areas we were hoping for. There were three software demonstrations.

Plenary Lectures:

George Andrews (Pennsylvania State University)  
“Search algorithms in the study of  $q$ -series”

Ken Ono (Pennsylvania State University and the University of Wisconsin at Madison)  
“Congruences for  $p(n)$  and some questions of Serre on the Fourier coefficients of modular forms”

Barry McCoy (Institute for Theoretical Physics, Stony Brook)  
“Rogers-Ramanujan identities in statistical mechanics and conformal field theory”

Doron Zeilberger (Temple University)  
“A tutorial on Mint: Akalu Tefera’s brilliant fully-automated implementation of the continuous multi-WZ method”

Sergei Suslov (Arizona State University)  
“Basic Fourier series: Introduction, analytic and numerical investigation”

Dennis Stanton (University of Minnesota)  
“Open problems in  $q$ -series”

The papers in this volume represent many of the topics covered at the conference. Although Bill Gosper and Mike Hirschhorn were unable to attend the conference, they were able to contribute papers to these proceedings. The order of articles is alphabetical by author.

We would like to thank the sponsors of our conference: the Institute for Fundamental Theory (University of Florida), the National Science Foundation, the National Security Agency, the UF Department of Mathematics and The Number Theory Foundation. We would also like to thank Denise Marks (University of South Florida) for typing some of the papers.

Frank G. Garvan  
University of Florida, Gainesville  
March 8, 2001.

Mourad E. H. Ismail  
University of South Florida, Tampa  
March 8, 2001.

## Participants

Scott Ahlgren<sup>\*†</sup> (Colgate University)  
Krishna Alladi<sup>†</sup> (University of Florida )  
George Andrews<sup>\*</sup> (Pennsylvania State University)  
Alexander Berkovich<sup>\*†</sup> (University of Florida)  
Bruce Berndt<sup>\*†</sup> (University of Illinois)  
Doug Bowman (University of Illinois )  
David Bradley<sup>\*</sup> (University of Maine)  
David Bressoud (Macalester College )  
John Brillhart<sup>\*</sup> (University of Arizona)  
Heng-Huat Chan<sup>\*</sup> (National University of Singapore)  
Youn-Seo Choi<sup>\*</sup> (Korean Advanced Institute of Science and Technology, Seoul)  
David and Gregory Chudnovsky<sup>‡</sup> (Polytechnic University)  
Charles Dunkl<sup>\*</sup> (University of Virginia)  
Dennis Eichhorn<sup>\*</sup> (University of Arizona )  
Frank Garvan (University of Florida )  
Ira Gessel<sup>\*</sup> (Brandeis University)  
Antonio Guerra (University of South Florida )  
Robert Gustafson<sup>\*</sup> (Texas A&M University)  
Mourad Ismail (University of South Florida )  
Soon-Yi Kang (University of Illinois )  
Marvin Knopp<sup>\*†</sup> (Temple University)  
Wolfram Koepf<sup>\*</sup> (HTWK, Leipzig)  
Christian Krattenthaler<sup>\*</sup> (Vienna University)  
Richard Lewis<sup>\*†</sup> (Sussex University)  
Zhi-Guo Liu<sup>\*†</sup> (Xinxiang Education College, P.R. China)  
Jeremy Lovejoy<sup>\*</sup> (Pennsylvania State University)  
Barry McCoy<sup>\*</sup> (Stony Brook)  
Richard McIntosh<sup>\*</sup> (University of Regina)  
Steve Milne<sup>\*†</sup> (Ohio State University)  
Maki Murata<sup>\*†</sup> (Pennsylvania State University)  
K.A. Muttalib<sup>\*</sup> (University of Florida)  
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Thomas Prellberg<sup>\*†</sup> (Syracuse University)  
Axel Riese<sup>\*†</sup> (RISC, Linz)  
Jaebum Sohn (University of Illinois )  
Dennis Stanton<sup>\*†</sup> (University of Minnesota)



Luz M. Suarez (University of South Florida )  
M.V. Subbarao\*<sup>†</sup> (University of Alberta)  
Sergei Suslov\* (Arizona State University)  
Akalu Tefera\* (Temple University )  
Rhiannon Weaver\* (Pennsylvania State University)  
Jinhee Yi (University of Illinois )  
G. Yoon (University of South Florida )  
Doron Zeilberger\* (Temple University)  
Liang-Chang Zhang\* (Southwest Missouri State University)

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\* Speaker. Links to abstracts of all talks are available at  
<http://www.math.ufl.edu/~frank/qsconf.html>

<sup>†</sup> Contributed paper to these proceedings.

<sup>‡</sup> David and Gregory Chudnovsky were unable to make it to the conference. Their talk *Orthogonal Polynomials and the Solution of the Pulse Width Modulation Problem*, was delivered by Mourad Ismail.