HIGHER DEGREE FORMS CONFERENCE

May 21 – 23, 2009

University of Florida, Gainesville, FL 32611

ABSTRACTS

Manjul Bhargava, Princeton University
9:30am - 10:30am, Thursday, May 21, and 9:30am - 10:30am, Friday, May 22

A Survey of Forms Associated to Prehomogeneous Vector Spaces
Part I. Algebraic correspondences
Part II. Applications to density and counting theorems
Quadratic forms have the special property that there is essentially just one (nondegenerate) quadratic form over $\mathbb{C}$ having n variables, up to isomorphism. This is one of the key properties of quadratic forms that makes their study very rich, both algebraically and analytically.

It is thus natural to look at other spaces of forms that share this special property over $\mathbb{C}$. Such spaces are known as *prehomogeneous vector spaces*. More precisely, a prehomogeneous vector space is a representation V of an algebraic group G which has a single open orbit over $\mathbb{C}$.

Prehomogeneous vector spaces turn out to conceal a great deal of algebraic and number-theoretic information, especially when studied over $\mathbb{Z}$. In the first lecture, we outline some of the various algebraic objects that are parametrized by prehomogeneous vector spaces.

In the second lecture, we discuss some recent analytic applications of the algebraic results described in the first lecture, particularly to counting number fields and to the proofs of certain cases and variations of the Cohen-Lenstra and Cohen-Martinet heuristics on class groups.

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Jaka Cimpric, University of Ljubljana, Slovenia
2:00pm - 2:30pm, Thursday, May 21

Higher product levels of skew fields
The n-th level of a field is the minimal number of elements such that the sum of their $2n$-th powers is equal to -1. I will briefly survey the commutative theory developed by Joly and Becker in the seventies and my relatively recent noncommutative extensions. For skew fields, several generalizations exist, because a product of two $2n$-th powers need not be a $2n$-th power any more.

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Benedict Gross, Harvard University
11:00AM - 12:00PM, FRIDAY, MAY 22

Restriction of representations of classical groups
I will define the classical groups and will discuss several restriction problems in their representation theory, including restriction of irreducible representations from $U(n)$ to $U(n-1)$. This work, which is joint with W.-T. Gan and D. Prasad, attempts to predict the multiplicities in the restriction from number-theoretic data of the Langlands parameters.

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Jonathan Hanke, University of Georgia
2:45PM - 3:15PM, THURSDAY, MAY 21

Class numbers and Mass formulas for quadratic forms
This talk will describe how to use exact mass formulas to determine quadratic forms of small class number, particularly those positive definite forms of class number one. The mass of a quadratic form connects the class number (i.e. number of classes in the genus) of a quadratic form with the volume of its adelic stabilizer, and is explicitly computable in terms of special values of zeta functions and their twists. Comparing this with known results about the sizes of automorphism groups one can make precise statements about the growth of the class number, and in principle determine those quadratic forms of small class number. These ideas can also be expressed more generally in the context of a linear algebraic group, and similar results should hold in that context as well.

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Wei Ho, Princeton University
2:00PM - 3:00PM, FRIDAY, MAY 22

Orbit parametrizations of curves
We discuss parametrizations of geometric data, such as curves with specified vector bundles, by orbits of certain special representations of algebraic groups. As quotients of affine spaces, these realizations of the moduli spaces can be quite explicit. We will examine a few of these parametrizations and their generalizations as well as possible links to classical algebraic geometry and number theory.

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Detlev Hoffmann, University of Nottingham
3:15PM - 4:15PM, FRIDAY, MAY 22

Differential forms and bilinear forms under field extensions
An important object in the algebraic theory of quadratic forms is the Witt ring of bilinear forms over a field. We are interested in the determination of the kernel of the restriction map from the Witt ring of a field to that of a field extension, the so-called Witt kernel for that field extension. For fields of characteristic not 2, Witt kernels are known only for a few types of field extensions.

We determine the Witt kernels in the case of characteristic 2 for all simple algebraic extensions and extensions given by function fields of hypersurfaces. A main tool in the proof concerns the module of differential forms over fields of characteristic two and its behavior under field extensions. We determine more generally the kernel of the restriction map between the module of differential forms over a field of arbitrary positive characteristic and that of an extension field given by a simple algebraic extension or a function field of a hypersurface. An important special case is that of the function field of a diagonal form of degree p in characteristic p.

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Igor Klep, University of Ljubljana, Slovenia
3:30pm - 4:00pm, Thursday, May 21

Central simple algebras with involution and positive polynomials
Consider a central simple algebra A with involution $*$. The involution is called *positive* if the involution trace form $\mathrm{tr}(x^*x)$ is positive semidefinite (w.r.t. a fixed ordering of the center). A symmetric element b is defined to be *positive* if the scaled involution trace form $\mathrm{tr}(x^*bx)$ is positive semidefinite giving rise to an *ordering* of the central simple algebra A. We discuss how these can be used to give a Positivstellensatz characterizing noncommutative real polynomials positive semidefinite on $d \times d$ matrices. This talk is partially based on joint work with T. Unger.

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Bruce Reznick, University of Illinois, Urbana-Champaign
11:00am - 12:00pm, Saturday, May 23

Recent results in 19th century algebra
The length of a form p of degree d is the minimum number of linear forms whose d-th powers sum to p. Two familiar theorems about quadratic forms is that the length cannot decrease if the base field is enlarged, and, in the real case, the Law of Inertia. We show that the length depends on the field when $d > 2$ in a non-trivial way and that the Law of Inertia holds for binary quartic forms, but fails for binary sextic forms. We will also present a mysterious identity involving quadratics taken to the 14-th power.

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Gordan Savin, University of Utah
11:00am - 12:00pm, Thursday, May 21

Integral structure of some internal modules
Let $P = MN$ be a maximal parabolic subgroup in a Chevalley group. The unipotent radical N has a natural filtration by subgroups such that the consecutive quotients are linear representations of M. These are called internal modules. The structure of these modules over the ring of integers can be very interesting and non-trivial as the work of Bhargava shows. In this lecture we shall give a description of these modules in the most banal case, when G is simply laced and N is abelian.

Takashi Taniguchi, Kobe University, Japan
2:45pm - 3:15pm, Saturday, May 23

Relations among Dirichlet series whose coefficients are class numbers of binary cubic forms
We study relations among Dirichlet series whose coefficients are class numbers of binary cubic forms in $\mathrm{SL}_2(\mathbb{Z})$-invariant lattices. We show that each of the Dirichlet series associated with the lattices satisfies a simple explicit relation to that of the dual lattices. These series are examples of zeta functions of prehomogeneous vector spaces and satisfy certain functional equations. As an application of our result, we rewrite the functional equation in a self dual form. This is joint work with Y. Ohno and S. Wakatsuki.

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Michael Volpato, University of California, San Diego
2:00pm - 2:30pm, Saturday, May 23

Hecke actions on Fourier coefficients of modular forms
After discussing classes of modular forms whose Fourier coefficients are indexed by classical algebraic structures, we describe the action of the spherical Hecke algebra on these Fourier coefficients explicitly, generalizing a well-known result of Hecke.

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Melanie Matchett Wood, Princeton University
9:30am - 10:30am, Saturday, May 23

Rings and ideals associated to binary forms
We can associate rings and ideals to binary forms of degree n, generalizing Gauss composition and the Delone-Faddeev/Gan-Gross-Savin parametrization of cubic rings by binary cubic forms. We can say explicitly what rings and ideals are parametrized by binary n-ic forms for all n. There is also a space of forms that parametrizes all ideal classes of the rings associated to binary forms. Finally, we can do this work not just over the integers, but over an arbitrary base ring or scheme.

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