Dyson's rank and the Andrews-Garvan crank to the moduli 8, 9 and 12.

Richard Lewis

The University of Sussex.

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Abstract

In 1944, Dyson **[Dys]** defined the rank of a partition as the difference between its largest part and the number of its parts. Denoting by $N(r, m, n)$ the number of partitions of n whose ranks are congruent to r modulo m, he noticed that a number of linear relations appeared to hold between the numbers $N(r, m, kn + s)$, when $(m, k) = (5, 5)$ and $(7, 7)$. These relations were all later proved by Atkin and Swinnerton-Dyer **[A+SD]**.

Some years later (1986-7), Andrews and Garvan **[A+G]** defined the crank of a partition. With $M(r, m, n)$ denoting the number of partitions of n whose cranks are congruent to r modulo m, Garvan **[Gar]**, **[Gar1, Gar2]** established a number of linear relations between the $M(r, m, kn + s)$, when $(m, k) = (5, 5)$, $(7, 7)$, $(11, 11)$, $(8, 4)$, $(9, 3)$ and $(10, 5)$.

I give here some relations between the $N(r, m, kn + s)$ when $(m, k) = (8, 4)$, $(9, 3)$ and $(12, 2)$. I also give some relations between the $N(r, m, kn + s)$ and the $M(r, m, kn + s)$ for $(m, k) = (4, 2)$, $(8, 4)$ and $(9, 3)$ and some inequalities holding amongst the $N(r, 9, 3n + s)$ and between the $N(r, 8, 4n + s)$ and the $M(r, 8, 4n + s)$.

All but one of the proofs of the main theorems use the theory of modular forms of half-integral weight on certain subgroups of $SL_2(\mathbb{Z})$ to show that each of the theorems may be proved simply by examining the first few cases. The same methods may be used to prove all the relations of Dyson and Garvan.

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