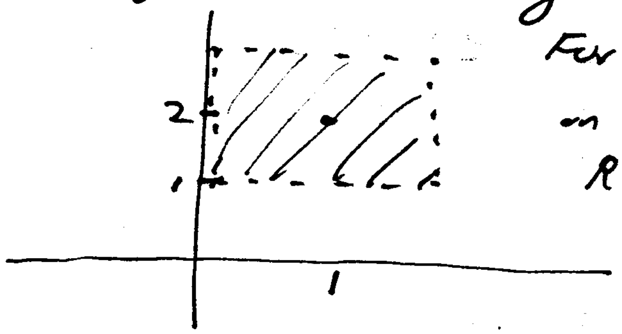
Hence, they are both continuous on an open rectangle R containing $(x_0,y_0)=(1,2)$.



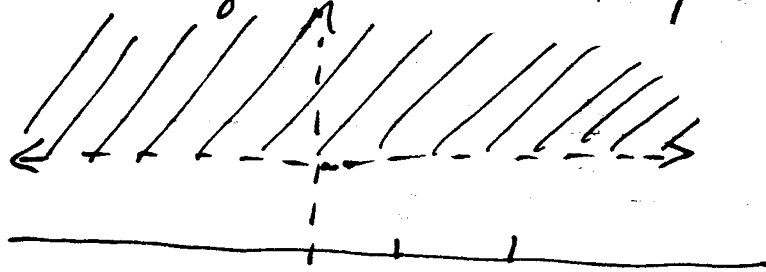
For instance on $R=\{(x,y):\ 0<x<2,\ 1<y<3\}$.

The Existence-Unique Thm implies the IVP has a unique solution $y=\phi(x)$ on some open interval $1-h<x<1+h$ containing $x=1$.

Example: Determine whether the Thm implies that the IVP

$$\frac{dy}{dx}=\frac{1}{x}-\sqrt{y-1}, \qquad y(2)=1$$

has a unique soln on an open interval containing $x=2$.



$f(x,y)=1/x-\sqrt{y-1}$

we have seen that $f(x,y)$ & $\frac{\partial f}{\partial y}$ are both continuous for (x,y) satisfying $\cancel{x\neq 0}$ & $y>1$.

$\frac{\partial f}{\partial y}$ is not continuous at $(1,2)$ & not continuous on any open rectangle containing $(1,2)$.

No conclusion can be drawn from the Thm.