Homework EX 1.2: 3, 5, 6, 9, 10, 11, 12, 14, 15, 19, 20, 23, 24, 25, 26, 27, 28.

1.2 Solutions and initial value problems

EXPLICIT SOLUTION

Example Show that the function $\phi(x) = x - 1 + e^{-x}$ is an explicit solution to the DE

$$\frac{dy}{dx} + y = x.$$

Soln: Let $y = x - 1 + e^{-x}$.

Then $\frac{dy}{dx} = 1 - e^{-x}$,

and

$$\frac{dy}{dx} + y = (1 - e^{-x}) + (x - 1 + e^{-x}) = x.$$

Hence $y = \phi(x) = x - 1 + e^{-x}$ is a solution to the DE.

WARNING: The following is not proper.

$$\frac{dy}{dx} + y = x$$

$$(1 - e^{-x}) + (x - 1 + e^{-x}) = x$$

$$x = x.$$

Example Show that for any constant c, $\phi(x) = x - 1 + ce^{-x}$ is an explicit soln to the DE

$$\frac{dy}{dx} + y = x.$$

Soln: Let c be any constant and let $y = x - 1 + ce^{-x}$.

Then $\frac{dy}{dx} = 1 - ce^{-x}$, and

$$\frac{dy}{dx} + y = (1 - ce^{-x}) + (x - 1 + ce^{-x}) = x.$$

Hence, $y = \phi(x) = x - 1 + ce^{-x}$ is an explicit soln.