IMPLICIT SOLUTIONS

A relation $G(x,y) = 0$ is said to be an <u>implicit solution</u> to a DE on an interval I if it defines one or more explicit solutions to a DE.

<u>Example</u>: Show that the relation

$$x^2 + y^2 = 1$$

implicitly defines a solution to the DE

$$\frac{dy}{dx} = \frac{-x}{y}.$$

<u>Method 1</u>: Suppose $x^2 + y^2 = 1$.

Then $y^2 = 1 - x^2$ and $y = \pm\sqrt{1-x^2}$.

$$\frac{dy}{dx} = \pm \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \pm \frac{-x}{\sqrt{1-x^2}}.$$

$$\frac{dy}{dx} = -x\left(\pm \frac{1}{\sqrt{1-x^2}}\right)$$

and $$\frac{dy}{dx} = \frac{-x}{y}.$$

<u>Method 2</u>: Suppose $y = y(x)$ and

$$x^2 + y^2 = 1.$$

Then $$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}(1),$$

$$2x + 2y\frac{dy}{dx} = 0,$$

$$y\frac{dy}{dx} = -x,$$

and $$\frac{dy}{dx} = \frac{-x}{y} \qquad (\text{assuming } y \neq 0).$$