

Example (#10) Assume the equation

$$y - \ln y = x^2 + 1$$

defines a function  $y = y(x)$  implicitly. Determine whether the function satisfies the DE

$$\frac{dy}{dx} = \frac{2xy}{y-1}$$

using implicit differentiation.

Soln: Suppose  $y = y(x)$  and

$$y - \ln y = x^2 + 1.$$

Then

$$\frac{d}{dx}(y - \ln y) = \frac{d}{dx}(x^2 + 1)$$

$$\frac{dy}{dx} - \frac{1}{y} \frac{dy}{dx} = 2x$$

$$y \frac{dy}{dx} - \frac{dy}{dx} = 2xy$$

$$(y-1) \frac{dy}{dx} = 2xy$$

$$\frac{dy}{dx} = \frac{2xy}{y-1}$$

(assuming  $y \neq 1$ ).

Hence the function satisfies the given D.E.