Initial value problems.

Example: (a) Show that $y = ce^{-x} + 1$ is a solution to

$$\frac{dy}{dx} + y = 1$$

for any constant c.

(b) Find a solution to the IVP

$$\frac{dy}{dx} + y = 1, \qquad y(0) = -3.$$

Soln: (a) Let $y = ce^{-x} + 1$. Then

$$\frac{dy}{dx} = -ce^{-x} \text{ and } \frac{dy}{dx} + y = -ce^{-x} + ce^{-x} = 1.$$

Hence y is a soln to $\frac{dy}{dx} + y = 1$.

(b)

$y = ce^{-x} + 1$. We want $y(0) = -3$.

$y(0) = c + 1 = -3, \quad c = -4.$

A solution is $y = -4e^{-x} + 1$.

Example

NOTE: For a first order IVP the initial condition has the form $y(x_0) = y_0$.

Example (a) Show that

$$y = c_1 \sin x + c_2 \cos x + e^{-x}$$

are solutions to $y'' + y = 2e^{-x}$ for any constants c_1, c_2.

(b) Find a solution to the IVP

$$y'' + y = 2e^{-x}, \quad y(0) = 2, \quad y'(0) = 1.$$