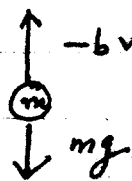


(1)

Chapter 2 First Order Differential Equations

2.1 Intro: Motion of a Falling body



Let $v(t)$ be the velocity (at time t) of a falling body.

Assume force due to air resistance is prop. to velocity. By Newton's second law

$$F = m \frac{dv}{dt}$$

So

$$m \frac{dv}{dt} = mg - bv$$

(b constant
 g grav. constant)

Then

$$m dv = (mg - bv) dt$$

$$\frac{m}{mg - bv} dv = dt$$

(separation of variables)

$$\int \frac{m}{mg - bv} dv = \int dt$$

$$-\frac{m}{b} \ln |mg - bv| = t + C_1$$

$$\ln |mg - bv| = -\frac{bt}{m} + \frac{mg}{b}$$

$$|mg - bv| = e^{-\frac{bt}{m} + \frac{mg}{b}} = e^{\frac{mg}{b}} e^{-bt/m}$$

$$mg - bv = \pm e^{\frac{mg}{b}} e^{-bt/m}$$

$$mg - bv = C e^{-bt/m}$$

$$bv = mg - C e^{-bt/m}$$

$$v = \frac{mg}{b} - \frac{C}{b} e^{-bt/m}$$

(general soln. of DE)