Chapter 2 First Order Differential Equations

2.1 Intro: Motion of a Falling body

(air resistance) Let $v(t)$ be the velocity (at time t) of a falling body.

Assume force due to air resistance is prop. to velocity. By Newton's second law

$$F = m\frac{dv}{dt}$$

so

$$m\frac{dv}{dt} = mg - bv \qquad (b \text{ constant}, g \text{ grav. constant})$$

Then

$$m\,dv = (mg - bv)\,dt$$

$$\frac{m}{mg-bv}\,dv = dt \qquad \text{(separation of variables)}$$

$$\int \frac{m}{mg-bv}\,dv = \int dt$$

$$-\frac{m}{b}\ln|mg-bv| = t + C_1$$

$$\ln|mg-bv| = -\frac{bt}{m} + \frac{mC_1}{b}$$

$$|mg-bv| = e^{-\frac{bt}{m}+\frac{mC_1}{b}} = e^{\frac{mC_1}{b}}e^{-bt/m}$$

$$mg-bv = \pm e^{mC_1/b}e^{-bt/m}$$

$$mg-bv = Ce^{-bt/m}$$

$$bv = mg - Ce^{-bt/m}$$

$$\boxed{v = \frac{mg}{b} - \frac{C}{b}e^{-bt/m}}$$

(general soln. of DE)