Example (#8) Solve

$$\frac{dy}{dx} = \frac{1}{xy^3}$$

$$y^3\,dy = \frac{1}{x}\,dx$$

$$\int y^3\,dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{4}y^4 = \ln|x| + C'$$

$$y^4 = 4\ln|x| + 4C'.$$

General soln is given by

$$y = \left(4\ln|x| + C\right)^{\frac{1}{4}}$$

Example (#14) Solve

$$\frac{dy}{dx} = 3x^2(1+y^2)$$

$$\frac{dy}{1+y^2} = 3x^2\,dx$$

$$\int \frac{dy}{1+y^2} = \int 3x^2\,dx$$

$$\tan^{-1}(y) = x^3 + C$$

$$y = \tan(x^3 + C)$$

~~$y^3 = \tan(x^3+C)$~~

~~$y = \pm\sqrt{\tan(x^3+C)}$~~ where C any constant.

$$y = \tan(x^3+c) \qquad (c \text{ any constant})$$