Example (#20)

Solve the IVP

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2y+1}, \qquad y(0) = -1.$$

$$2y+1\ dy = (3x^2+4x+2)\,dx$$

$$\int (2y+1)\,dy = \int (3x^2+4x^2+2)\,dx$$

$$y^2+y = x^3+2x^2+2x+C$$

$y(0) = -1 \Rightarrow$

$$1-1 = 0+C \quad \& \quad C=0.$$

So soln is given implicitly by

$$y^2+y = x^3+2x^2+2x$$

Note we can solve this eqn for y by completing the square.

$$y^2+y+\frac{1}{4} = x^3+2x^2+2x+\frac{1}{4}$$

$$\left(y+\frac{1}{2}\right)^2 = x^3+2x^2+2x+\frac{1}{4}$$

$$y+\frac{1}{2} = \pm\sqrt{x^3+2x^2+2x+\frac{1}{4}}$$

$y(0) = -1 \Rightarrow \quad y+\frac{1}{2} = -1+\frac{1}{2} = -\frac{1}{2}$

and

$$y+\frac{1}{2} = -\sqrt{x^3+2x^2+2x+\frac{1}{4}}$$

$$\boxed{y = -\frac{1}{2} - \sqrt{x^3+2x^2+2x+\frac{1}{4}}}$$