

MAPLE

(4A)

> with(DEtools);

> DE := diff(y(x), x) = (3*x^2 + 4*x + 2)/(2*y(x) + 1);

$$DE := \frac{dy(x)}{dx} = \frac{3x^2 + 4x + 2}{2y(x) + 1}$$

> odeadvisor(DE);
[- separable]

(Maple advises
that this DE
is separable)

For more help try

> odeadvisor(DE, help);

Formal Justification of Method

Consider the DE:

$$(*) \quad f(y) \frac{dy}{dx} = g(x) \quad \Leftrightarrow \quad \frac{dy}{dx} = \frac{g(x)}{f(y)}$$

$$\text{Let } F(y) = \int f(y) dy \quad \Leftrightarrow \quad F'(y) = f(y) \\ \text{and } G(x) = \int g(x) dx \quad \Leftrightarrow \quad G'(x) = g(x).$$

Now assume $y = y(x)$.

$$\frac{d}{dx} F(y(x)) = F'(y(x)) \frac{dy}{dx} = f(y(x)) \frac{dy}{dx}$$

Hence (*) is equivalent to

$$\frac{d}{dx} F(y(x)) = \frac{d}{dx} G(x)$$

$$\Leftrightarrow F(y(x)) = G(x) + C.$$