Example Solve $\frac{dy}{dx} = x^2 y$.

Soln:

$$y = c\, e^{\int x^2 dx} = c\, e^{x^3/3}.$$

The general soln is given by

$$y = c\, e^{x^3/3}$$

where c is any constant.

Now we are ready to tackle the general soln of a linear DE

$$a(x) \frac{dy}{dx} + b(x) y = c(x)$$

$$\Leftrightarrow \quad \frac{dy}{dx} + \frac{b(x)}{a(x)} y = \frac{c(x)}{a(x)} \qquad (\text{assuming } a(x) \neq 0)$$

$$\Leftrightarrow \quad \frac{dy}{dx} + p(x) y = q(x) \qquad [\text{STANDARD FORM}]$$

where $p(x) = \frac{b(x)}{a(x)}$, $q(x) = \frac{c(x)}{a(x)}$.

Method for solving a 1[st] order linear DE:

(1) Get the DE in standard form:

$$(*) \quad \frac{dy}{dx} + p(x) y = q(x).$$

(2) Multiply both sides of $(*)$ by an integrating factor $\mu(x)$

$$\mu(x) \frac{dy}{dx} + \mu(x) p(x) y = q(x) \mu(x)$$

So that the LHS is

$$\frac{d}{dx} \mu(x) y = \mu(x) \frac{dy}{dx} + \mu'(x) y$$

ie We want $\mu'(x) = p(x)\mu(x)$

ie take $\mu(x) = e^{\int p(x) dx}$.