(3) The DE can be written as

$$\frac{d}{dx}\,\mu(x)\,y = q(x)\,\mu(x), \quad \text{where } \mu(x) = e^{\int p(x)dx}$$

Integrate both sides to ~~and~~ obtain

$$\mu(x)\,y = \int q(x)\,\mu(x)\,dx + C$$

(4) The general soln is given by

$$\boxed{y = \frac{1}{\mu(x)}\left\{\int q(x)\mu(x)\,dx + C\right\},}$$

where C is any constant.

Example: Solve

$$\frac{dy}{dx} - \frac{y}{x} = 2x+1 \qquad (\text{for } x>0).$$

Here $p(x) = -1/x$, $q(x) = 2x+1$.

STEP 1: Let $\mu(x) = e^{\int p(x)dx} = e^{\int -1/x\,dx}$
$= e^{-\ln x} = \frac{1}{x}$.

STEP 2: Let $z = \mu(x)\,y = \frac{1}{x}y$.

Then

$$\frac{dz}{dx} = \frac{1}{x}\frac{dy}{dx} + y\left(-\frac{1}{x^2}\right)$$

$$= \frac{1}{x}\left(\frac{dy}{dx} - \frac{1}{x}y\right)$$

$$= \frac{1}{x}(2x+1) = 2 + \frac{1}{x}.$$

$$\frac{dz}{dx} = 2 + \frac{1}{x}.$$