STEP 3 $z = \int 2 + \frac{1}{x}\,dx + C$

$= 2x + \ln x + C$

STEP 4 $z = y/x$ and so $y = xz$ and the general soln of the DE is given by

$$\boxed{y = x(2x + \ln x + C)} \quad \text{for } x > 0$$

where C is any constant.

Example (p. 52) A rock contains two radioactive isotopes RA_1, RA_2; RA_1 decays into RA_2 which in turn decays into stable atoms. Assume rate at which RA_1 decays into RA_2 is $50e^{-10t}$ kg/sec. Let $y(t)$ be the mass of RA_2 present at time t. The rate of decay of RA_2 is prop. to $y(t)$.

$\frac{dy}{dt}$ = rate of creation − rate of decay

$$\frac{dy}{dt} = 50e^{-10t} - ky \qquad (k > 0).$$

Assume $k = 2$ and $y(0) = 40$ kg. Find $y(t)$.

$$\frac{dy}{dt} + 2y = 50e^{-10t}$$

Let $\mu = e^{\int 2\,dt} = e^{2t}$.

Let $z = \mu y = e^{2t} y$.

Then $\frac{dz}{dt} = e^{2t}\frac{dy}{dt} + 2e^{2t}y$

$= e^{2t}\left(\frac{dy}{dt} + 2y\right)$

$= e^{2t}\, 50e^{-10t}$

$= 50e^{-8t}$