$$z = \int 50 e^{8t}\, dt$$

$$z = \frac{50 e^{8t}}{8} + C$$

$$e^{2t} y = \frac{25 e^{8t}}{4} + C$$

$y(0) = 40 \Rightarrow \quad 40 = \frac{25}{4} + C$

$$C = 40 + \frac{25}{4} = \frac{160+25}{4} = \frac{185}{4}.$$

$$y = e^{-2t}\left(\frac{185}{4} - \frac{25}{4} e^{8t}\right)$$

$$y = \frac{185}{4} e^{-2t} - \frac{25}{4} e^{6t}, \quad \text{for } t \geq 0.$$

Example (#22)

$$\sin x \frac{dy}{dx} + y \cos x = x \sin x \qquad y(\pi/2) = 2$$

for $0 < x < \pi$,

$$\frac{dy}{dx} + \cot x\, y = x.$$

Let $\mu(x) = e^{\int \frac{\cos x}{\sin x} dx}$

$$= e^{\ln(\sin x)} = \sin x.$$

Let $z = \mu(x) y = \sin x\, y.$

$$\frac{dz}{dx} = \sin x \frac{dy}{dx} + y \cos x = x \sin x.$$

So $z = \int \underbrace{x}_{u} \underbrace{\sin x\, dx}_{dv} \qquad \left(v = \int \sin x\, dx = -\cos x\right)$

$$= uv - \int v\, du = -x \cos x - \int -\cos x\, dx$$

$$= -x \cos x + \sin x + C.$$