

(3)

Example (#16)

Find a soln to IVP

$$y'' - 4y' + 3y = 0, \quad y(0) = 1, \quad y'(0) = \frac{1}{3}.$$

A.E:  $r^2 - 4r + 3 = 0$   
 $(r-1)(r-3) = 0$   
 $r = 1, 3.$

Two solns are

$$y_1 = e^t, \quad y_2 = e^{3t}.$$

Let  $y = c_1 e^t + c_2 e^{3t}, \quad c_1, c_2 \in \mathbb{R}.$

Then  $y$  is a soln of the DE.

$$y' = c_1 e^t + 3c_2 e^{3t}.$$

$$y(0) = c_1 + c_2 = 1$$

$$y'(0) = c_1 + 3c_2 = \frac{1}{3}$$

$$2c_2 = -\frac{2}{3}, \quad c_2 = -\frac{1}{3}$$

$$c_1 = 1 - c_2 = \frac{4}{3}.$$

So

$$y = \frac{4}{3} e^t - \frac{1}{3} e^{3t} \quad \text{is a soln to the IVP.}$$

NOTE: It can be shown that this is the only soln.