Let $y_3 = k y_1$ where $k = \dfrac{y_2(\tau)}{y_1(\tau)}$.

Then y_3 is a soln of $(*)$ because y_1 is.

$$y_3(\tau) = k y_1(\tau) = \frac{y_2(\tau)}{y_1(\tau)} y_1(\tau) = y_2(\tau).$$

$$y_3'(\tau) = k y_1'(\tau) = \frac{y_2(\tau)}{y_1(\tau)} y_1'(\tau) = \frac{y_2'(\tau) y_1(\tau)}{y_1(\tau)} = y_2'(\tau).$$

Hence y_3 & y_2 are solns of the IVP

$$ay'' + by' + cy = 0 \qquad y(\tau) = y_2(\tau),\ y'(\tau) = y_2'(\tau).$$

By uniqueness,

$$y_3 = y_2,$$

$$y_2 = k y_1 \quad \&$$

y_1, y_2 are linearly dependent.

<u>Other Cases</u> See text.

<u>NOTE</u>: The quantity $y_1(\tau) y_2'(\tau) - y_1'(\tau) y_2(\tau)$ is called the WRONSKIAN of y_1, y_2 at τ.

<u>Theorem</u> Let $a, b, c \in \mathbb{R}$, $a \neq 0$. Let $Y_0, Y_1 \in \mathbb{R}$, $t_0 \in I$. Let y_1, y_2 be two independent solns to

$$(*) \qquad ay'' + by' + cy = 0 \qquad \text{on } I.$$

(1) The general soln to $(*)$ is $y = c_1 y_1 + c_2 y_2$ where c_1, c_2 are any constants.

(2) There exist unique constants c_1, c_2 such that $y_0 = c_1 y_1 + c_2 y_2$ is soln to the IVP

$$ay'' + by' + cy = 0, \quad y(t_0) = Y_0, \quad y'(t_0) = Y_1.$$