<u>Definition</u> Let y_1, y_2 be two d'ble (differentiable) functions. The wronskian of y_1, y_2 is defined by

$$W[y_1, y_2](t) = y_1(t) y_2'(t) - y_1'(t) y_2(t)$$

$$= \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix}$$

<u>Example</u> Let $y_1 = e^t$, $y_2 = e^{2t}$.

$$W[y_1, y_2] = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^t & e^{2t} \\ e^t & 2e^{2t} \end{vmatrix}$$

$$= 2e^{3t} - e^{3t} = e^{3t}$$

<u>Theorem</u>: Suppose y_1, y_2 are two linearly dependent d'ble functions on an open interval I Then

$$W[y_1, y_2](t) = 0 \qquad \text{for all } t \in I.$$

<u>Proof</u>: Suppose y_1, y_2 are dependent (& d'ble on I). So one is a constant mult. of other, say $y_2 = c y_1$, some constant c.

$$W[y_1, y_2](t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} y_1 & cy_1 \\ y_1' & cy_1' \end{vmatrix} = c y_1 y_1' - c y_1 y_1' = 0.$$

<u>NOTE</u>: The converse of this theorem is not true. See for example #35 in Ex 4.2.