

(8)

Suppose r is a root of the auxiliary eqn

$$ar^2 + br + c = 0$$

(a ≠ 0)

$$(*) \quad r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Then $y = e^{rt}$ is a soln to

$$(**) \quad ay'' + by' + cy = 0.$$

Theorem

- (1) If $\Delta = b^2 - 4ac > 0$ then (*) has two distinct real roots r_1, r_2 ,
 $y_1 = e^{r_1 t}$, $y_2 = e^{r_2 t}$ are two linearly indept. solns to (**) & the general soln of (**) is

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}, \text{ where}$$

c_1, c_2 are any constants.

- (2) If $\Delta = b^2 - 4ac = 0$ then (*) has a double root

$$r = -b/2a,$$

$$y_1 = e^{-\frac{b}{2a}t}, \quad y_2 = t e^{-\frac{b}{2a}t} \text{ are two}$$

indept. solns of (**) & the general soln of (**) is

$$y = (c_1 + c_2 t) e^{-\frac{b}{2a}t}, \text{ where } c_1, c_2 \text{ any constants.}$$