<u>Proof of (2)</u> Suppose $\Delta = b^2 - 4ac = 0$,
$b^2 = 4ac$.

We know
$$y_1 = e^{-bt/(2a)}$$ is a sol'n of (**).

Let $y_2 = t e^{-\frac{bt}{2a}}$.

Then
$$y_2' = t\left(-\frac{b}{2a}\right) e^{-\frac{bt}{2a}} + e^{-\frac{bt}{2a}}$$
$$y_2'' = t\left(\frac{b^2}{4a^2}\right) e^{-\frac{bt}{2a}} - \frac{b}{2a} e^{-\frac{b}{2a}} - \frac{b}{2a} e^{-\frac{bt}{a}}$$

$$a y_2'' + b y_2' + c y_2 = e^{-\frac{bt}{2a}}\left(t\left(\frac{b^2}{4a} - \frac{b^2}{2a} + c\right) - b + b\right)$$
$$= 0$$

since $\frac{b^2}{4a} - \frac{b^2}{2a} + c = \frac{b^2}{4a} - \frac{b^2}{2a} + \frac{b^2}{4a} = \frac{b^2}{2a} - \frac{b^2}{2a} = 0.$

Hence $y_2 = t e^{-\frac{bt}{2a}}$ is also a sol'n of (**).

y_1, y_2 are clearly indep't since

$\frac{y_2}{y_1} = t$ is not constant. The result follows.

<u>Example</u> Solve
$$y'' + 6y' + 9y = 0.$$

AE: $r^2 + 6r + 9 = 0$
$(r+3)^2 = 0$
$r = -3$ is a double root.

The general sol'n is
$$y = (c_1 + c_2 t) e^{-3t},$$ where c_1, c_2 any constants.