4.3 Auxiliary Equations with Complex Roots

Suggested HW: odds 1–25

Review:

A.E: $ar^2 + br + c = 0.$

Let $\Delta = b^2 - 4ac.$

If $\Delta < 0$ then the A.E. has two complex roots

$$r = \alpha \pm i\beta$$

where $\alpha = \frac{-b}{2a}$, $\beta = \frac{\sqrt{4ac - b^2}}{2a}$, $i = \sqrt{-1}$, $i^2 = -1$.

Example Find the roots of

$$r^2 - 4r + 7 = 0.$$

$$r = \frac{4 \pm \sqrt{16 - 28}}{2}$$

$$= 2 \pm \sqrt{\left(\frac{-12}{4}\right)} = 2 \pm \sqrt{-3}$$

$$= 2 \pm \sqrt{3}\, i.$$

Roots are $r = 2 + \sqrt{3}i$ & $r = 2 - i\sqrt{3}$.

Definition of e^{rt} when $r = \alpha + i\beta$

$$e^{rt} = e^{(\alpha + i\beta)t} = e^{\alpha t + i\beta t}$$

$$= e^{\alpha t} e^{i\beta t}$$

When x is real

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{i\beta t} := 1 + \frac{i\beta t}{1} + \frac{(i\beta t)^2}{2!} + \frac{(i\beta t)^3}{3!} + \frac{(i\beta t)^4}{4!} + \cdots$$

$$= 1 + i\beta t - \frac{\beta^2 t^2}{2!} - i\frac{\beta^3 t^3}{3!} + \frac{\beta^4 t^4}{4!} + \cdots$$