$$= (2+i\sqrt{3})\, e^{2t} \cos\sqrt{3}t \quad + (2i-\sqrt{3}) e^{2t} \sin\sqrt{3}t$$

$$= (2+i\sqrt{3})\, e^{2t} \cos\sqrt{3}t \quad + i(2+i\sqrt{3}) e^{2t} \sin\sqrt{3}t$$

$$= (2+i\sqrt{3})(e^{2t} \cos\sqrt{3}t + i\, e^{2t} \sin\sqrt{3}t)$$

$$= (2+i\sqrt{3})\, e^{(2+i\sqrt{3})t}.$$

<u>Theorem</u>: Let $r = \alpha + i\beta$ $(\alpha, \beta \in \mathbb{R})$.
Then $\frac{d}{dt} e^{rt} = r e^{rt}$.

<u>Example</u> Find two independent real solns of

$$y'' - 4y' + 7y = 0.$$

Ae: $r^2 - 4r + 7 = 0$, has roots
$r = 2 \pm \sqrt{3}i$.

Two complex solutions are

$$z_1 = e^{(2+\sqrt{3}i)t}$$
$$= e^{2t}(\cos\sqrt{3}t + i \sin\sqrt{3}t),$$
$$z_2 = e^{(2-\sqrt{3}i)t}$$
$$= e^{2t} (\cos(-\sqrt{3}t) + i \sin(-\sqrt{3}t))$$
$$= e^{2t} (\cos\sqrt{3}t - i \sin\sqrt{3}t)$$

Two real solns are

$$y_1 = \tfrac{1}{2}(z_1 + z_2) = e^{2t} \cos\sqrt{3}t$$

$$y_2 = \tfrac{1}{2i}(z_1 - z_2) = e^{2t} \sin\sqrt{3}t$$

y_1, y_2 are two linearly indept. solns so general soln is

$$y = c_1 e^{2t} \cos\sqrt{3}t + c_2 e^{2t} \sin\sqrt{3}t$$

$$= e^{2t} (c_1 \cos(\sqrt{3}t) + c_2 \sin(\sqrt{3}t))$$

where c_1, c_2 are any constants.