

(13)

Theorem Suppose $\Delta = b^2 - 4ac < 0$.

The A.L. $ar^2 + br + c = 0$ has two complex
sols $r = \alpha \pm i\beta$ where $\alpha = \frac{-b}{2a}$, $\beta = \frac{\sqrt{4ac-b^2}}{2a}$.
The DE

$ay'' + by' + cy = 0$
has two linearly independent solns

$$y_1 = e^{\alpha t} \cos \beta t, \quad y_2 = e^{\alpha t} \sin \beta t$$

and the general soln is given by

$$y = e^{\alpha t} (C_1 \cos \beta t + C_2 \sin \beta t),$$

where C_1, C_2 are any constants.