4.4 Nonhomogeneous Equations: The Method of Undetermined Coeffs

Suggested HW: odds 1-31

The method of undetermined coefficients is a method for finding a particular soln of a nonhomogeneous linear DE with constant coeffs when RHS has a special form.

Example (#1c) Find a particular soln y_p of

$$y'' + 2y' - y = 10.$$

Try $y_p = c$. Then $y_p' = 0$, $y_p'' = 0$ &

$y_p'' + 2y_p' - y_p = -c = 10$ & $c = -10$.

So

$y_p = -10$ is a particular soln.

Example Find a particular soln y_p to

$$y'' - 2y' + y = 3t^3 - 18t^2 + 20t - 9.$$

Let

$$\begin{aligned} y_p &= at^3 + bt^2 + ct + d \\ y_p' &= 3at^2 + 2bt + c \\ y_p'' &= 6at + 2b \end{aligned}$$

$$\begin{aligned} y_p'' &= 6at + 2b \\ -2y_p' &= -6at^2 - 4bt - 2c \\ +y_p &= at^3 + bt^2 + ct + d \end{aligned}$$

$$= at^3 + (b-6a)t^2 + (c-4b+6a)t + (d-2c+2b)$$

$$= 3t^3 - 18t^2 + 20t - 9.$$

$a = 3$. $b - 6a = b - 18 = -18$, $b = 0$.

$c - 4b + 6a = c + 18 = 20$, $c = 2$.

$d - 2c + 2b = d - 4 = -9$, $d = -5$.

So $y_p = 3t^3 + 2t - 5$ is a particular soln.