We consider 2^{nd} order linear nonhomogeneous DEs with constant coeffs:

$$ay'' + by' + cy = g(x)$$

Type (I) $g(x)$ is a polynomial of degree n.

(a) If $c \neq 0$, then

$$y_p = A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x + A_0.$$

(b) If $c = 0$, $b \neq 0$ then

$$y_p = x(A_n x^n + \dots + A_1 x + A_0).$$

(c) If $c = 0$, $b = 0$, then

$$y_p = x^2(A_n x^n + \dots + A_1 x + A_0).$$

Type (II) $g(x) = p_n(x) e^{\alpha x}$ (where $p_n(x)$ is a polynomial of degree n).

(a) If $r = \alpha$ is a root of the aux. eqn. (ie $e^{\alpha x}$ is a soln of the homog. eqn) then

$$y_p = e^{\alpha x}(A_n x^n + \dots + A_1 x + A_0).$$

Example Find a particular soln of

$$y'' + 2y' + y = e^{2x}(9x - 21).$$

Clearly $r = 2$ is not a root of the aux. eqn. $r^2 + 2r + 1 = 0$.

So let $y_p = e^{2x}(Ax + B)$.

$$y_p' = e^{2x} A + 2(Ax + B) e^{2x}$$
$$= e^{2x}(A + 2B + 2Ax)$$
$$y_p'' = e^{2x}(2A) + (A + 2B + 2Ax) e^{2x}(2)$$
$$= e^{2x}(4A + 4B + 4Ax).$$

$$y_p'' + 2y_p' + y_p = e^{2x}(4A + 4B + 4Ax + 2A + 4B + 4Ax + B + Ax)$$
$$= e^{2x}(6A + 9B + 9Ax) = e^{2x}(9x - 21)$$

$6A + 9B = -21$

$9A = 9$

$A = 1$. $9B = -21 - 6A = -27$, $B = -3$