So $\quad y_p = e^{2x}(x-3)$ is a particular sol$^{\underline{n}}$.

(b) $r=\alpha$ is a single root of the aux. eq^n.
(ie: $y=e^{\alpha x}$ is a ~~used~~ sol$^{\underline{n}}$ of the homog. eq^n but $y=xe^{\alpha x}$ is not) then

$$y_p = xe^{\alpha x}(A_n x^n + \cdots + A_1 x + A_0).$$

Example Find the form of a particular sol$^{\underline{n}}$ to

$$y'' + y' - 2y = e^x(x^2+1)$$

A.E: $\quad r^2+r-2=0$

$(r+2)(r-1)=0$ has roots $r=1,-2$.

$r=1$ is a single root. So the form of

$$y_p = xe^x(Ax^2+Bx+C).$$

(c) $r=\alpha$ is a double root of the aux. eq^n
(ie $y=e^{\alpha x}$, $y=xe^{\alpha x}$ are sol^{ns} to the homog. de), then

$$y_p = x^2 e^{\alpha x}(A_n x^n + \cdots + A_1 x + A_0).$$

Type (III)

$$g(x) = p_n(x)\, e^{\alpha x}\cos\beta x + q_m(x)\, e^{\alpha x}\sin\beta x$$

where $p_n(x)$ is a poly of deg. n & $q_m(x)$ is a poly of deg. m. Let $N=\max(n,m)$.

(a) If $r=\alpha+i\beta$ is not a root of the aux. eq^n then

$$y_p = P_N(x)\, e^{\alpha x}\cos\beta x + Q_N(x)\, e^{\alpha x}\cos\beta x$$

where $P_N(x), Q_N(x)$ are generic polynomials of degree N.

(b) if $r=\alpha+i\beta$ is a root of the aux. eq^n. Then

$$y_p = x\left(P_N(x)\, e^{\alpha x}\cos\beta x + Q_N(x)\, e^{\alpha x}\sin\beta x\right).$$