4.5 The Superposition Principle & Undetermined Coeffs Revisited

Sug. HW odds 1–35, 41

Theorem Suppose y_1, y_2 are two indept. solns of the homogeneous DE

$$(*) \quad ay'' + by' + cy = 0.$$

Suppose y_p is a particular solution to the nonhomog DE

$$(**) \quad ay'' + by' + cy = f(t) \qquad (\text{for } t \in I).$$

Then the general soln to $(**)$ is

$$y = y_p + c_1 y_1 + c_2 y_2,$$

where c_1, c_2 are any constants. Further, the IVP

$$(***) \quad ay'' + by' + cy = f(t), \quad y(t_0) = Y_0, \quad y'(t_0) = Y_1$$

has a unique solution.

Proof in part

$$ay_1'' + by_1' + cy_1 = 0$$
$$ay_2'' + by_2' + cy_2 = 0$$
$$ay_p'' + by_p' + cy_p = f(t).$$

Let $y = y_p + c_1 y_1 + c_2 y_2$ (c_1, c_2 constants).

Then

$$ay'' + by' + cy = a(y_p'' + c_1 y_1'' + c_2 y_2'') + b(y_p' + c_1 y_1' + c_2 y_2') + c(y_p + c_1 y_1 + c_2 y_2)$$

$$= ay_p'' + by_p' + cy_p + c_1(ay_1'' + by_1' + cy_1) + c_2(ay_2'' + by_2' + cy_2)$$

$$= f(t) + c_1 0 + c_2 0 = f(t).$$

Hence $y = y_p + c_1 y_1 + c_2 y_2$ is a soln of $(**)$.