Conversely, suppose $\hat{y}$ is a soln to (**).

Let $y = \hat{y} - y_p$. Then

$$ay'' + by' + cy = a(\hat{y}'' - y_p'') + b(\hat{y}' - y_p') + c(\hat{y} - y_p)$$

$$= a\hat{y}'' + b\hat{y}' + c\hat{y} - (ay_p'' + by_p' + cy_p)$$

$$= f(t) - f(t) = 0.$$

So $\hat{y} - y_p$ is a soln to (*) &

$$\hat{y} - y_p = c_1 y_1 + c_2 y_2 \quad \text{some constants } y_1, y_2,$$

&

$$\hat{y} = y_p + c_1 y_1 + c_2 y_2.$$

Hence the general soln of (**) is

$$y = y_p + c_1 y_1 + c_2 y_2$$

where c_1, c_2 are any constants. $\square$.

<u>Example</u> Find the general soln of

$$y'' + 2y' + y = e^{2x}(9x - 21).$$

We know (from the previous section (see p. 15)) that one particular soln is $y_p = e^{2x}(x-3)$.

A.E: $r^2 + 2r + 1 = 0$

$(r+1) = 0$

$r = -1$ is a double root.

So $y_1 = e^{-x}$, $y_2 = xe^{-x}$ are 2 lin. indept. solns of the homog. DE. Hence the general soln of the nonhom DE is

$$y = e^{2x}(x-3) + c_1 e^{-x} + c_2 x e^{-x},$$

where c_1, c_2 are any constants.