SUPERPOSITION PRINCIPLE

Suppose y_1 is a solution to

(1) $ay'' + by' + cy = g_1(t)$,

and y_2 is a solution to

(2) $ay'' + by' + cy = g_2(t)$.

Then $y = c_1 y_1 + c_2 y_2$ (c_1, c_2 constant)

is a soln to

(3) $ay'' + by' + cy = c_1 g_1(t) + c_2 g_2(t)$.

Proof:

$ay_1'' + by_1' + cy_1 = g_1(t)$ since y_1 is a soln of (1).

$ay_2'' + by_2' + cy_2 = g_2(t)$ ... y_2 ..... (2).

$a(c_1y_1 + c_2y_2)'' + b(c_1y_1 + c_2y_2)' + c(c_1y_1 + c_2y_2)$

$= a(c_1y_1'' + c_2y_2'') + b(c_1y_1' + c_2y_2') + c(c_1y_1 + c_2y_2)$

$= c_1(ay_1'' + by_1' + cy_1)$

$+ c_2(ay_2'' + by_2' + cy_2)$

$= c_1 g_1(t) + c_2 g_2(t)$ & $c_1y_1 + c_2y_2$ is a soln of (3). $\square$

Example We are given that

$y_1 = \frac{1}{4}\sin 2x$ is a soln to $y'' + 2y' + 4y = \cos 2x$

$y_2 = \frac{x}{4} - \frac{1}{8}$ is a soln to $y'' + 2y' + 4y = x$.

Find a soln to $y'' + 2y' + 4y = 2x - 3\cos 2x$,

$= -3\cos 2x + 2x$.

By superposition principle,

$y = -3y_1 + 2y_2$

$= \frac{3}{4}\sin 2x + 2(x/4 - 1/8)$

$= -3/4 \sin 2x + x/2 - 1/4$

is a soln to $y'' + 2y' + 4y = 2x - 3\cos 2x$.