<u>Example</u> (#28) Solve the IVP

(*) $y'' + y' - 12y = e^t + e^{2t} - 1, \; y(0) = 1, \; y'(0) = 3.$

A.E: $r^2 + r - 12 = 0$

$(r-3)(r+4) = 0$

Roots are $r = 3, -4$.

A particular soln of $y''+y'-12y = e^t$ has form $y_p = Ae^t$ since $r=1$ not a root of A.E.

A particular soln of $y''+y'-12y = e^{2t}$ has form $y_p = Be^{2t}$ since $r=2$ is not a root of A.E.

A particular soln of $y''+y'-12y = -1$ has form $y_p = C$ since $-12 \neq 0$.

By superposition, a particular soln of (*) has form

$$y_p = Ae^t + Be^{2t} + C.$$

$$y_p' = Ae^t + 2Be^{2t}$$

$$y_p'' = Ae^t + 4Be^{2t}$$

$$y_p'' + y_p' - 12y_p = -10Ae^t - 6Be^{2t} - 12C = e^t + e^{2t} - 1.$$

$$A = -1/10, \quad B = -1/6, \quad C = +1/12.$$

so

$$y_p = -\frac{1}{10}e^t - \frac{1}{6}e^{2t} + \frac{1}{12}.$$

$r = 3, -4$ so $y_1 = e^{3t}$, $y_2 = e^{-4t}$ are indept. solns of the homog. DE. Hence the general soln of (*) is

$$y = c_1 e^{3t} + c_2 e^{-4t} - \frac{1}{10}e^t - \frac{1}{6}e^{2t} + 1/12.$$

$$y(0) = c_1 + c_2 - 1/10 - 1/6 + 1/12 = 1$$

$$c_1 + c_2 = 1 + 1/10 + 1/6 - 1/12 = \frac{60+6+10-5}{60} = \frac{71}{60}$$

$$y'(t) = 3c_1 e^{3t} - 4c_2 e^{-4t} - 1/10\, e^t - 1/3\, e^{2t}$$

$$y'(0) = 3c_1 - 4c_2 - 1/10 - 1/3 = 3$$