

4.6 Variation of Parameters

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Variation of parameters is a method for finding a particular soln $y_p(t)$ of a nonhom. linear DE

provided we know two linear indep. soln of $y_1(t), y_2(t)$ of the corresponding homog. DE

$$ay'' + by' + cy = 0.$$

We assume

$$y_p(t) = v_1(t) y_1 + v_2(t) y_2$$

$$y_p' = v_1 y_1' + v_1' y_1 + v_2 y_2' + v_2' y_2$$

We assume

$$v_1' y_1 + v_2' y_2 = 0.$$

So
and

$$y_p' = v_1 y_1' + v_2 y_2',$$

$$y_p'' = v_1 y_1'' + v_1' y_1' + v_2 y_2'' + v_2' y_2'$$

Therefore,

$$ay_p'' + by_p' + cy_p = \begin{matrix} \boxed{c v_1 y_1} & + & \boxed{c v_2 y_2} \\ & + & \\ & \boxed{b v_1 y_1'} & + & \boxed{b v_2 y_2'} \\ & + & \\ & \boxed{a v_1 y_1''} & + & \boxed{a v_2 y_2''} \\ & + & a v_1' y_1' & + & a v_2' y_2' \end{matrix}$$

$$= a v_1' y_1' + a v_2' y_2' =$$

Since $ay_1'' + by_1' + cy_1 = 0$

& $ay_2'' + by_2' + cy_2 = 0.$

Hence, we want

$$\boxed{\begin{matrix} y_1 v_1' + y_2 v_2' = 0 \\ y_1' v_1' + y_2' v_2' = \frac{g}{a} \end{matrix}}$$

Solve these equations for v_1', v_2' . Then find v_1, v_2 & y_p .