Example (Final Exam Sp. 2000) (#5)

Find the general sol$^{\underline{n}}$ of the DE

$$y'' - 2y' + y = \frac{e^t}{t}, \quad \text{for } t > 0.$$

* First we solve the homog. DE

$$y'' - 2y' + y = 0.$$

A.E: $r^2 - 2r + 1 = 0$

$$(r-1)^2 = 0$$

$r = 1$ is a double root.

Hence $y_1 = e^t$, $y_2 = te^t$ are two lin. indept. sol$^{\underline{n}}$s.

* We use variation of parameters to find a particular sol$^{\underline{n}}$.

Let

$$y_p = v_1 y_1 + v_2 y_2 = v_1 e^t + v_2 e^t t.$$

We solve

$$y_1 v_1' + y_2 v_2' = 0$$

$$y_1' v_1' + y_2' v_2' = \frac{g}{a} = \frac{e^t}{t}$$

$$e^t v_1' + te^t v_2' = 0$$

$$e^t v_1' + (e^t t + e^t) v_2' = \frac{e^t}{t}$$

$$e^t v_2' = e^t/t, \quad v_2' = 1/t.$$

So $v_2 = \int \frac{1}{t}\, dt$ & $v_2 = \ln t$.

$$v_1' = -tv_2' = -1, \quad \& \ v_1 = -t.$$

Hence $y_p = -te^t + (\ln t)\, te^t$,

and the general sol$^{\underline{n}}$ is given by

$$y = c_1 e^t + c_2 te^t - te^t + (\ln t) te^t,$$

for c_1, c_2 any constants.