

(30)

Critically damped ($b^2 = 4mk$).

(31)

The a.e. has a double root $r = -b/2m$.

The general soln is

$$y(t) = (c_1 + c_2 t) e^{-(b/2m)t}$$

Overdamped ($b^2 > 4mk$)

The A.E. has 2 real roots

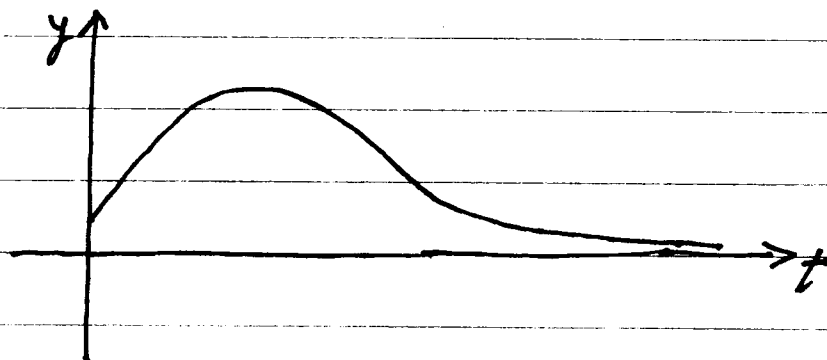
$$r_1 = \frac{-b}{2m} - \frac{\sqrt{b^2 - 4mk}}{2m} < 0$$

$$r_2 = \frac{-b}{2m} + \frac{\sqrt{b^2 - 4mk}}{2m} < 0 \quad \left(\text{since } \sqrt{b^2 - 4mk} < \sqrt{b^2} = b \right)$$

The general soln

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

$$\lim_{t \rightarrow \infty} y(t) = 0.$$

NOTE: Assume $(c_1, c_2) \neq (0, 0)$.

(i) The mass crosses the equilibrium point at most once.

Suppose note $r_1 < r_2 < 0$.

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t} = e^{r_1 t} (c_1 + c_2 e^{(r_2 - r_1)t}) = 0$$

$$\text{when } c_1 + c_2 e^{(r_2 - r_1)t} = 0$$

$$\text{ie } e^{(r_2 - r_1)t} = -c_1/c_2 \quad (\text{assuming } c_2 \neq 0)$$

This has one soln if $-c_1/c_2 > 0$ ie $t = \frac{1}{(r_2 - r_1)} \ln(-c_1/c_2)$
 and no soln if $-c_1/c_2 \leq 0$.