#11 A 1kg mass is attached to a spring with stiffness 100N/m. The damping constant for the system is 0.2 Nsec/m. If the mass is pulled 5m beyond the equil. position with a velocity of 1m/sec when will it attain its max. displacement to the right?

Let $y(t)$ = displacement (to right) from equilibrium position in m.

Then $my'' + by' + ky = 0$

$1y'' + 0.2y' + 100y = 0$ $\qquad 4ac = 4 \cdot 5 \cdot 500$

A.E. $r^2 + 0.2r + 100 = 0$

$5r^2 + r + 500 = 0$

$$r = \frac{-1 \pm \sqrt{1 - 10000}}{10}$$

$$r = -\frac{1}{10} \pm \frac{\sqrt{9999}}{10} = \frac{-1}{10} \pm \frac{3}{10}\sqrt{1111}$$

$$y = e^{-t/10}\left(c_1 \cos\beta t + c_2 \sin\beta t\right), \quad \beta = \tfrac{3}{10}\sqrt{1111}$$

Given $y(0) = 0$, $y'(0) = 1$.

$y(0) = c_1 = 0.$

$y = e^{-t/10} c_2 \sin\beta t$

$y'(t) = c_2 \sin\beta t \left(\frac{-1}{10} e^{-t/10}\right) + \beta c_2 \cos\beta t\, e^{-t/10}$

$y'(0) = \beta c_2 = 1, \quad c_2 = \frac{1}{\beta}.$

So

$$y'(t) = e^{-t/10}\left(\frac{-1}{10\beta}\sin\beta t + \cos\beta t\right).$$

Max. displ. occurs when $y'(t) = 0$, since $y(0) = 0$.

i.e. $\frac{1}{10\beta}\sin\beta t = +\cos\beta t$

$$\tan\beta t = +10\beta = +3\sqrt{1111}$$