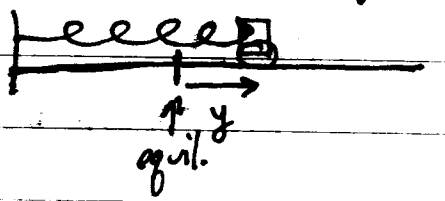
4.9 Forced Mechanical Vibrations

Homework: 3, 9, 11, 13.

Assume there is an additional external force $F(t)$ acting on a spring mass system.



$$F = ma = -ky - by'(t) + F(t)$$

($-ky$: F_{spring}; $-by'(t)$: $F_{damping}$)

$$my''(t) = -ky(t) - by'(t) + F(t)$$

$$(*)\quad my'' + by'(t) + ky(t) = F(t)$$

(non homog linear DE with constant coeffs).

Assume $F(t) = F_0 \cos \gamma t$ and $0 < b^2 < 4mk$ so that the system is under damped. The general soln of (*) has the form

$$y = y_p + y_h$$

where y_p is a particular soln & y_h is the general soln of the corresponding homogeneous DE. We found (in 4.8) that

$$y_h = Ae^{\alpha t} \sin(\beta t + \phi),$$

where $\alpha = \frac{-b}{2m}$, $\beta = \frac{\sqrt{4mk - b^2}}{2m}$, and A, ϕ are constants. The roots of A.E. are $r = \alpha \pm i\beta$. So $r = \gamma i$ is not a root of A.E. & y_p has the form

$$y_p = A_1 \cos \gamma t + A_2 \sin \gamma t$$
$$y_p' = -\gamma A_1 \sin \gamma t + \gamma A_2 \cos \gamma t$$
$$y_p'' = -\gamma^2 A_1 \cos \gamma t - \gamma^2 A_2 \sin \gamma t$$