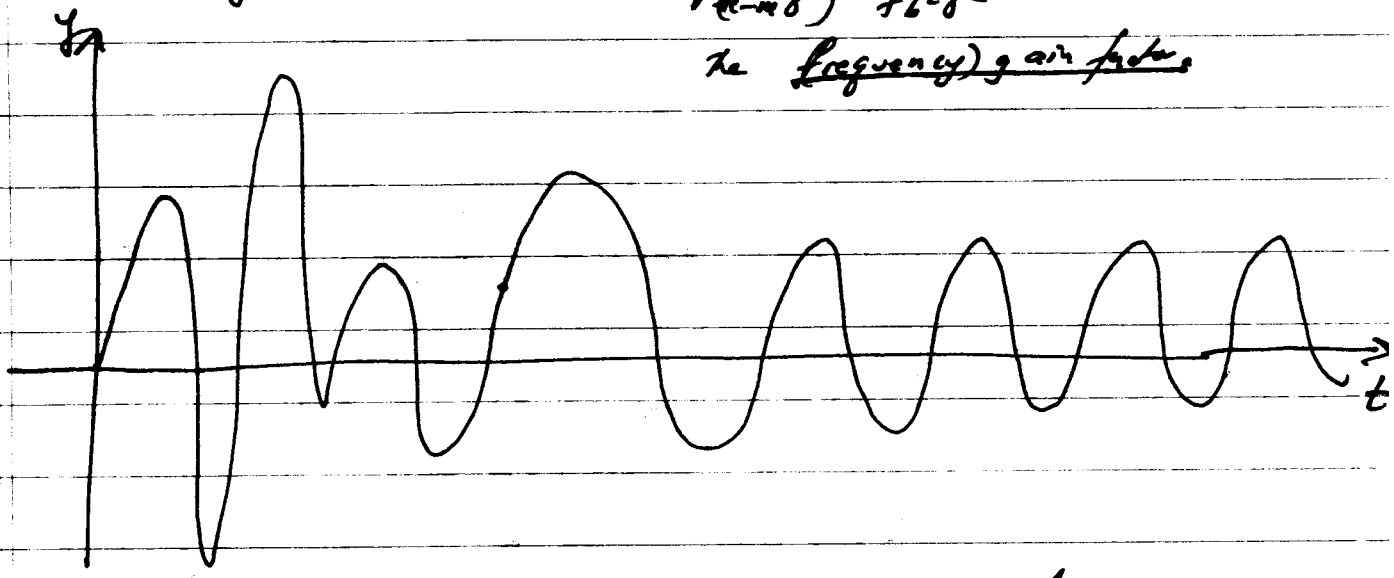
So

$$y = \underbrace{A e^{\alpha t} \sin(\beta t + \phi)}_{y_h} + \underbrace{\frac{F_0}{\sqrt{(k - m\gamma^2)^2 + b^2\gamma^2}} \sin(\gamma t + \theta)}_{y_p}$$

y_h damped oscillation and $y_h \to 0$ as $t \to \infty$ TRANSIENT PART of SOLUTION

y_p STEADY-STATE SOLUTION

The factor $\dfrac{1}{\sqrt{(k - m\gamma^2)^2 + b^2\gamma^2}}$ is called the frequency gain factor



Example Find the steady state soln of a spring system with $m = 1$ kg, $b = 2$ N sec/m, $k = 2$ N/m, and external force $F(t) = 10 \cos 2t$.

We want a particular soln to

$$y'' + 2y' + 2y = 10 \cos 2t.$$

Note $b^2 = 4$ $4mk = 4 \cdot 1 \cdot 2 = 8$, $b^2 < 4mk$ and the system is under-damped.

Method 1 $r^2 + 2r + 2 = 0$

$$(r+1)^2 = -1$$

$$r + 1 = \pm i, \quad r = -1 \pm i.$$

$r = 2i$ is not a soln so of A.E.