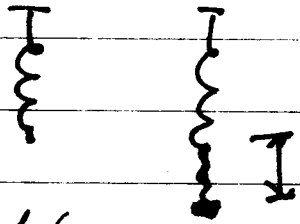
#12 A 2kg mass is attached to a spring hanging from the ceiling causing the spring to stretch 20cm upon coming to rest at equilibrium. At time $t=0$ the mass is displaced a further 5cm below equilibrium position and released. At this instant an external force $F(t) = 0.3 \cos t$ N is applied to the system. If the damping constant is 5 N s/m determine the eqn of motion of the system. What is the resonance frequency for the system.



$$mg = k\Delta L$$
$$2 \cdot 9.8 = k \cdot \frac{1}{5}$$
$$k = 10 \cdot 98 = 98$$

Let $y(t)$ = displacement below equilibrium.

Then $my'' + by' + ky = F(t)$

$$2y'' + 5y' + 98y = 0.3 \cos t$$

$y(0) = 5\text{cm} = \frac{5}{100} = 0.05$ m $\quad y'(0) = 0$.

$b^2 = 25 \quad 4mk = 4 \cdot 2 \cdot 98$

The system is underdamped since $b^2 < 4mk$.

Also $b^2 < 2mk$ so the system has a resonance frequency.

$$\gamma_r = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}} = \sqrt{\frac{98}{2} - \frac{25}{2 \cdot 4}} = \sqrt{\frac{367}{8}} \approx 6.773$$

$$\text{Resonance frequency} = \frac{\gamma_r}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{367}{8}} \approx 1.078$$