

The A.E. of this DE is

(5)

$$3r^2 + 8r + 4 = 0$$

$$(3r + 2)(r + 2) = 0$$

$$r = -2, -\frac{2}{3}$$

$$\therefore I_{2,h} = c_1 e^{-2t} + c_2 e^{-\frac{2}{3}t}$$

$r=0$ is not a root of the A.E. so a particular soln

$$\text{has the form } I_{2,p} = A, \quad 4A = 2, \quad A = \frac{1}{2}.$$

$\therefore$

$$I_2 = \frac{1}{2} + c_1 e^{-2t} + c_2 e^{-\frac{2}{3}t}$$

$$I_2(0) = \frac{1}{2} + c_1 + c_2 = 0$$

$$\dot{I}_2 = -2c_1 e^{-2t} - \frac{2}{3}c_2 e^{-\frac{2}{3}t}$$

$$\dot{I}_2(0) = -2c_1 - \frac{2}{3}c_2 = 0$$

$$c_1 + c_2 = -\frac{1}{2}$$

$$-2c_1 - \frac{2}{3}c_2 = 0$$

$$2c_1 + 2c_2 = -1$$

$$\frac{4}{3}c_2 = -1$$

$$c_2 = -\frac{3}{4}$$

$$c_1 = \frac{1}{4}$$

and

$$I_2 = \frac{1}{2} + \frac{1}{4}e^{-2t} - \frac{3}{4}e^{-\frac{2}{3}t}$$

$$I_3 = 4I_2 + 3\dot{I}_2$$
$$= 2 - \frac{1}{2}e^{-2t} - \frac{3}{2}e^{-\frac{2}{3}t}$$

$$I_1 = I_2 + I_3$$

$$= \frac{5}{2} - \frac{1}{4}e^{-2t} - \frac{7}{4}e^{-\frac{2}{3}t}$$