6.3 THE ANNIHILATOR METHOD

Def$\underline{n}$ A linear differential operator L annihilates a function $f(x)$ if

$$L[f](x) = 0.$$

Example The differential operator $L = D - 5$ annihilates the function $f(x) = e^{5x}$

since

$$L[f] = D(e^{5x}) - 5e^{5x} = 5e^{5x} - 5e^{5x} = 0.$$

NOTE Here $D = \frac{d}{dx}$.

#22 (p. 362)

Use the annihilator method to find the form of a particular sol$\underline{n}$ y_p to the DE

$$(*) \quad y'' + 6y' + 8y = e^{3x} - \sin x.$$

We want a differential operator that annihilates $e^{3x} - \sin x$.

$D - 3$ annihilates e^{3x} &

$D^2 + 1$ annihilates $-\sin x$ & so

$(D-3)(D^2+1)$ annihilates e^{3x} & $-\sin x$ & hence $e^{3x} - \sin x$.

Suppose y_p is a particular sol$\underline{n}$ of $(*)$. Then

$$(D^2 + 6D + 8)[y_p] = e^{3x} - \sin x$$

$$(D+2)(D+4)[y_p] = e^{3x} - \sin x$$

&

$$(D-3)(D^2+1)(D+2)(D+4)[y_p] = (D-3)(D^2+1)(e^{3x} - \sin x) \;\&$$

$$(D-3)(D^2+1)(D+2)(D+4)[y_p] = 0.$$

also $(D+2)(D+4)[y_p] \neq 0$ so (30)
we see that y_p has the form
$$y_p = c_1 e^{3x} + c_2 \cos x + c_3 \sin x$$
for some constants c_1, c_2, c_3.

\# 30 (p. 362)

(*) $y''' + 2y'' - y' - 2y = e^x - 1.$

We want a differential operator that annihilates $e^x - 1$.

$D-1$ annihilates e^x &

D annihilates 1. So $D(D-1)$ annihilates e^x & 1 & so $e^x - 1$.

S'pose y_p is a soln to (*).

Then
$$(D^3 + 2D^2 - D - 2)[y_p] = (D^2(D+2) - (D+2))[y_p]$$
$$= (D^2 - 1)(D+2)[y_p] = e^x - 1.$$
So that
$$(D-1)\,D\,(D-1)(D+1)(D+2)[y_p] = D(D-1)[e^x - 1] \;\&$$
$$D(D-1)^2(D+1)(D+2)[y_p] = 0.$$
We also want $(D-1)(D+1)(D+2)[y_p] \neq 0$.

We see that the form of y_p is given by
$$y_p = c_1 + c_2 x e^x$$
for some constants c_1, c_2.