Chapter 7 Laplace Transforms

7.2 Definition of Laplace Transform (Sug Hw: odds 1–29)

Let $f(t)$ be a function defined for $t \geq 0$ (ie on the interval $[0,\infty)$). The *Laplace transform* of f denoted by $\mathcal{L}\{f\}$ is the function

$$F(s) := \int_0^\infty e^{-st} f(t)\,dt$$

$$= \lim_{N\to\infty} \int_0^N e^{-st} f(t)\,dt$$

provided the limit exists.

We write $\mathcal{L}\{f\} = \int_0^\infty e^{-st} f(t)\,dt$.

Example Find $\mathcal{L}\{1\}$ using the definition.

$$\int_0^N e^{-st}\,dt = \left.\frac{e^{-st}}{-s}\right]_0^N \qquad (\text{if } s \neq 0)$$

$$= \frac{e^{-sN}}{-s} + \frac{e^0}{s} = \frac{1}{s} - \frac{e^{-sN}}{s} = \frac{1}{s} - \frac{1}{s e^{sN}}.$$

$$\lim_{N\to\infty} \int_0^N e^{-st}\,dt = \lim_{N\to\infty} \frac{1}{s} - \frac{1}{s e^{sN}}$$

$$= \frac{1}{s} - 0 \qquad (\text{provided } s > 0).$$

$$= \frac{1}{s}. \quad \text{So } \int_0^\infty e^{-st}\cdot 1\,dt = \mathcal{L}\{1\} = 1/s \text{ for } s>0.$$

Hence

$$\mathcal{L}\{1\} = \frac{1}{s} \quad \text{for } s > 0.$$