<u>Example</u> Determine whether $f(t) = t\cos t$ has exponential ~~and~~ order.

$$|f(t)| = |t\cos t| = |t|\,|\cos t| \leqslant |t|$$

for all t, since $0 \leqslant |\cos t| \leqslant 1$.

$$\lim_{t\to\infty} \frac{t}{e^t} \qquad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{t\to\infty} \frac{1}{e^t} \qquad \text{(by L'H's rule)}$$

$$= 0.$$

Hence $\left|\frac{t}{e^t}\right| \leqslant 1$ eventually,

and there is a $T>0$:

$$|t| \leqslant e^t \quad \text{for } t \geqslant T$$

& $\quad |f(t)| \leqslant |t| \leqslant e^t \quad$ for $t \geqslant T$.

So $f(t)$ has exponential order $\alpha = 1$.

[Note: It can be shown that $t \leqslant e^t$ for all t.
See Bonus Prob. 4. $f(t) = t\cos t$ has exponential order α for any $\alpha > 0$.]

<u>Example</u> Determine whether $f(t) = e^{t^2}$ has exponential order. Let α be a constant.

$$~~\text{Now}~~ \lim_{t\to\infty} \frac{e^{t^2}}{e^{\alpha t}} = \lim_{t\to\infty} e^{t^2-\alpha t} = +\infty$$

$$\text{since } \lim_{t\to\infty} t^2 - \alpha t = \lim_{t\to\infty} t(t-\alpha) = +\infty.$$

Hence $f(t)$ can not be of exponential order α, for any α. So $f(t)$ is not of exponential order.