

Example Find $\mathcal{L}\{t \sin 2t\}$.

$$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2 + 2^2} = \frac{2}{s^2 + 4}, \text{ for } s > 0.$$

$$\mathcal{L}\{t \sin 2t\} = -\frac{d}{ds} \frac{2}{s^2 + 4} = (-)(-)(s^2 + 4)^{-2} \cdot 2(2s)$$

$$= \frac{4s}{(s^2 + 4)^2} \text{ for } s > 0.$$

Let $F(s) = \mathcal{L}\{f\}$.

$$\mathcal{L}\{t^2 f(t)\} = \mathcal{L}\{t(t f(t))\}$$

$$= -\frac{d}{ds} \mathcal{L}\{t f(t)\}$$

$$= -\frac{d}{ds} \left[-\frac{d}{ds} \mathcal{L}\{f\} \right]$$

$$= \frac{d^2}{ds^2} \mathcal{L}\{f\}.$$

In general,

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \mathcal{L}\{f\}.$$