7.3 Properties of The Laplace Transform

Sug HW: odds 1–25

Theorem: Suppose $\mathcal{L}\{f\}(s) = F(s)$ for $s > \alpha$.
Then $\mathcal{L}\{e^{at} f(t)\}(s) = F(s-a)$ for $s > \alpha + a$.

Proof:

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt \quad \text{for } s > \alpha.$$

$$\mathcal{L}\{e^{at} f(t)\} = \int_0^\infty e^{-st} e^{at} f(t)\, dt$$

$$= \int_0^\infty e^{-(s-a)t} f(t)\, dt$$

$$= F(s-a) \quad \text{if } s - a > \alpha \text{ or } s > \alpha + a. \quad \square$$

Example: Find $\mathcal{L}\{e^{2t} \sin 3t\}$.

We know

$$\mathcal{L}\{\sin 3t\} = \frac{3}{s^2 + 3^2} \quad \text{for } s > 0.$$

So $$\mathcal{L}\{e^{2t} \sin 3t\} = \frac{3}{(s-2)^2 + 9} \quad \text{for } s - 2 > 0$$

$$= \frac{3}{s^2 - 4s + 13} \quad \text{for } s > 2.$$