<u>Theorem</u> Let $f(t)$ be CTS on $[0,\infty)$ & $f'(t)$ be piecewise CTS on $[0,\infty)$ with both of exponential order α.
Then $\mathcal{L}\{f'\}(s) = s\,\mathcal{L}\{f\}(s) - f(0)$,
for $s > \alpha$.

<u>Proof</u>:

$$\int \underbrace{e^{-st}}_{u}\, \underbrace{f'(t)\,dt}_{dv} = uv - \int v\, du$$

$$= e^{-st} f(t) - \int f(t)\,(-s e^{-st})\,dt$$

$$= e^{-st} f(t) + s \int e^{-st} f(t)\,dt$$

$$\int_0^N e^{-st} f'(t)\,dt = e^{-st} f(t)\Big]_0^N + s\int_0^N e^{-st} f(t)\,dt$$

$$= e^{-sN} f(N) - f(0) + s\int_0^N e^{-st} f(t)\,dt$$

$$\lim_{N\to\infty} e^{-sN} f(N) = 0 \quad \text{for } s > \alpha$$

since $\quad |f(N)| \le M e^{-s\alpha} \quad$ for $N \geq T$

$$|e^{-sN} f(N)| \le M e^{-(s-\alpha)N}.$$

Hence

$$\mathcal{L}\{f'\} = \int_0^\infty e^{-st} f'(t)\,dt = -f(0) + s\int_0^\infty e^{-st} f(t)\,dt$$

$$= -f(0) + s\,\mathcal{L}\{f\}. \quad \square$$