Let $F(s) = \mathcal{L}\{f\}$ for $s > \alpha$.

$$\mathcal{L}\{f'\} = sF(s) - f(0).$$

$$\mathcal{L}\{f''\} = s\mathcal{L}\{f'\} - f'(0)$$
$$= s(sF(s) - f(0)) - f'(0)$$
$$= s^2F(s) - sf(0) - f'(0).$$

$$\mathcal{L}\{f'''\} = s\mathcal{L}\{f''\} - f''(0)$$
$$= s(s^2F(s) - sf(0) - f'(0)) - f''(0)$$
$$= s^3F(s) - s^2f(0) - sf'(0) - f''(0).$$

In general,

$$\mathcal{L}\{f^{(n)}\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$$

for $s > \alpha$ provided $f, f', \ldots, f^{n-1}$ are CTS on $[0,\infty)$, $f^{(n)}$ is piecewise CTS on $[0,\infty)$ & all functions are of exponential order α.

__Example__ Let y be a solution of

$$y'' + b^2y = 0. \qquad b>0 \text{ a constant.}$$

Find $\mathcal{L}\{y\}$ assuming transform exists.

Let $Y(s) = \mathcal{L}\{y\}$.

$$\mathcal{L}\{y''\} = s^2Y(s) - sy(0) - y'(0)$$
$$\mathcal{L}\{b^2y\} = b^2\mathcal{L}\{y\} = b^2Y(s)$$
$$\mathcal{L}\{y'' + b^2y\} = \mathcal{L}\{0\} = 0$$
$$(b^2 + s^2)Y(s) - s(y)(0) - y'(0) = 0$$

So